Institute: Jnan ch. Ghosh Polytechnic

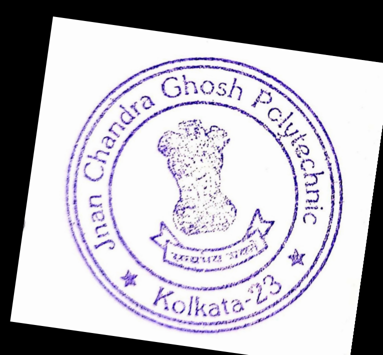
Deptt.-Mechanical Engineering.

Subject: Theory of Mechanics & Mechanism.

Year & Semester: 2nd year & S-4

Teacher: Ratan Sikdar

Topic discussed: Governors.



18.1. Introduction

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load *e.g.* when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

A little consideration will show, that when the load increases, the configuration of the governor changes and a valve is moved to increase the supply of the working fluid ; *conversely,* when the load decreases, the engine speed increases and the governor decreases the supply of working fluid.

Note : We have discussed in Chapter 16 (Art. 16.8) that the function of a flywheel in an engine is entirely different from that of a governor. It controls the speed variation caused by the fluctuations of the engine turning moment during each cycle of operation. It does not control the speed variations caused by a varying load. The varying demand for power is met by the governor regulating the supply of working fluid.

18.2. Types of Governors

The governors may, broadly, be classified as

1. Centrifugal governors, and 2. Inertia governors.

The centrifugal governors, may further be classified as follows :

- Centrifugal governors
  - Pendulum type
    - Watt governor
  - Loaded type
    - Dead weight governors
      - Porter governor
      - Proell governor
    - Spring controlled governors
      - Hartnell governor
      - Hartung governor
      - Wilson-Hartnell governor
      - Pickering governor

18.3. Centrifugal Governors

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the *controlling force**. It consists of two balls of equal mass, which are attached to the arms as shown in Fig. 18.1. These balls are known as *governor balls* or *fly balls*. The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle ; but can slide up and down. The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops *S*,*S* are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls.

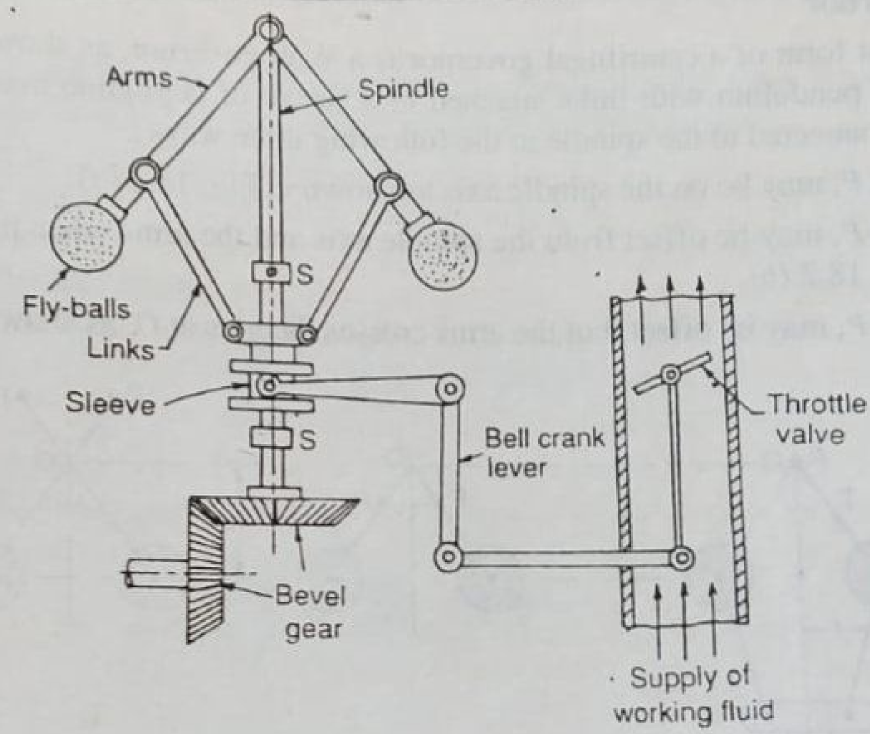


Fig. 18.1. Centrifugal governor.

When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced.

Note : When the balls rotate at uniform speed, controlling force is equal to the centrifugal force and they balance each other.

18.4. Terms Used in Governors

The following terms used in governors are important from the subject point of view ;

*1. *Height of a governor.* It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by h.

2. *Equilibrium speed.* It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

3. *Mean equilibrium speed.* It is the speed at the mean position of the balls or the sleeve.

4. *Maximum and minimum equilibrium speeds.* The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

Note : There can be many equilibrium speeds between the mean and the maximum and the mean and the minimum equilibrium speeds.

5. *Sleeve lift.* It is the vertical distance which the sleeve travels due to change in equilibrium speed.

18.5. Watt Governor

The simplest form of a centrifugal governor is a Watt governor, as shown in Fig. 18.2. It is basically a conical pendulum with links attached to a sleeve of negligible mass. The arms of the governor may be connected to the spindle in the following three ways :

1. The pivot P, may be on the spindle axis as shown in Fig. 18.2 (*a*).

2. The pivot P, may be offset from the spindle axis and the arms when produced intersect at O, as shown in Fig. 18.2 (*b*).

3. The pivot P, may be offset, but the arms crosses the axis at O, as shown in Fig. 18.2 (*c*).

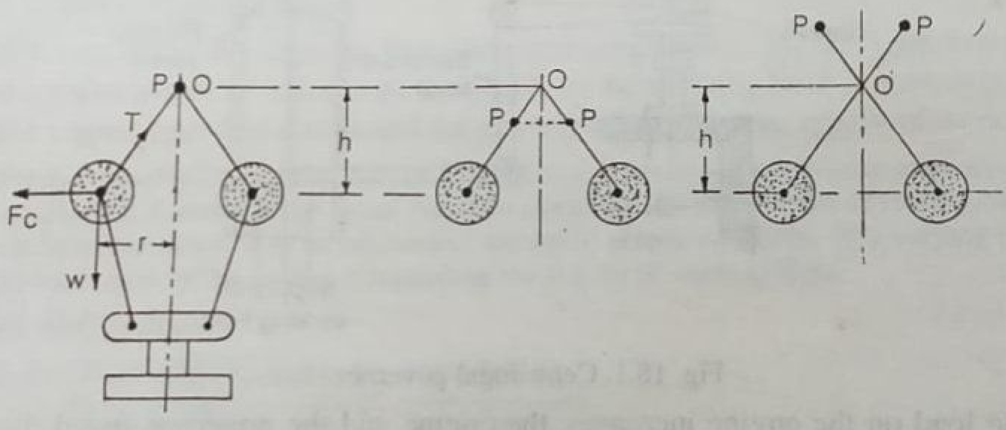


Fig. 18.2. Watt governor.

Let m = Mass of the ball in kg,

w = Weight of the ball in newtons = $m.g$,

T = Tension in the arm in newtons,

ω = Angular velocity of the arm and ball about the spindle axis in rad/s,

r = Radius of the path of rotation of the ball *i.e.* horizontal distance from the centre of the ball to the spindle axis in metres,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$, and

h = Height of the governor in metres.

It is assumed that the weight of the arms, links and the sleeve are negligible as compared to the weight of the balls. Now, the ball is in equilibrium under the action of

1. the centrifugal force (F_C) acting on the ball, 2. the tension (T) in the arm, and 3. the weight (w) of the ball.

Taking moments about point O, we have

$$F_C \times h = w \times r = m.g.r$$

or
$$m.\omega^2.r.h = m.g.r \quad \text{or} \quad h = g/\omega^2 \qquad \ldots (i)$$

When g is expressed in m/s^2 and ω in rad/s, then h is in metres. If N is the speed in r.p.m., then

$$\omega = 2\pi N/60$$

$$\therefore \quad h = \frac{9.81}{(2\pi N/60)^2} = \frac{895}{N^2} \text{ metres} \qquad \ldots (\because g = 9.81 \text{ m/s}^2) \quad \ldots (ii)$$

Note : We see from the above expression that the height of a governor h, is inversely proportional to N^2. Therefore at high speeds, the value of h is small. At such speeds the change in the value of h corresponding to a small change in speed is insufficient to enable a governor of this type to operate the mechanism to give the necessary change in the fuel supply. This governor may only work satisfactorily at relatively low speeds *i.e.* from 60 to 80 r.p.m.

Example 18.1. *Calculate the vertical height of a Watt governor when it rotates at 60 r.p.m. Also find the change in vertical height when its speed increases to 61 r.p.m.*

Solution. Given : $N_1 = 60$ r.p.m. ; $N_2 = 61$ r.p.m.

Initial height

We know that initial height,

$$h_1 = \frac{895}{(N_1)^2} = \frac{895}{(60)^2} = 0.248 \text{ m}$$

Change in vertical height

We know that final height,

$$h_2 = \frac{895}{(N_2)^2} = \frac{895}{(61)^2} = 0.24 \text{ m}$$

$\therefore$ Change in vertical height

$$= h_1 - h_2 = 0.248 - 0.24 = 0.008 \text{ m} = 8 \text{ mm} \quad \textbf{Ans.}$$

18.6. Porter Governor

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig. 18.3 (*a*). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any pre-determined level.

Consider the forces acting on one-half of the governor as shown in Fig. 18.3 (*b*).

Let
m = Mass of each ball in kg,

w = Weight of each ball in newtons = $m.g$,

M = Mass of the central load in kg,

W = Weight of the central load in newtons = $M.g$,

r = Radius of rotation in metres,

h = Height of governor in metres,

N = Speed of the balls in r.p.m.,

ω = Angular speed of the balls in rad/s = $2\pi N/60$ rad/s,

F_C = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$,

T_1 = Force in the arm in newtons.

T_2 = Force in the link in newtons.

α = Angle of inclination of the arm (or upper link) to the vertical, and

β = Angle of inclination of the link (or lower link) to the vertical.

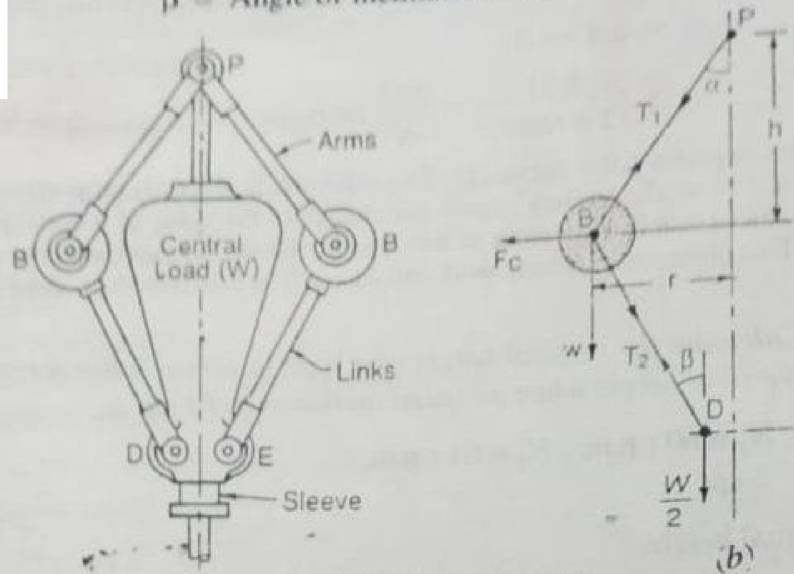


Fig. 18.3. Porter governor.

Though there are several ways of determining the relation between the height of the governor (h) and the angular speed of the balls (ω), yet the following two methods are important from the subject point of view :

1. Method of resolution of forces ; and 2. Instantaneous centre method.

1. *Method of resolution of forces*

Considering the equilibrium of the forces acting at D, we have

$$T_2 \cos\beta = \frac{W}{2} = \frac{M.g}{2} \quad \text{or} \quad T_2 = \frac{M.g}{2\cos\beta} \qquad \ldots(i)$$

Again, considering the equilibrium of the forces acting on B. The point B is in equilibrium under the action of the following forces, as shown in Fig. 18.3 (b).

(i) The weight of ball ($w = m.g$),

(ii) The centrifugal force (F_C).

(iii) The tension in the arm (T_1), and

(iv) The tension in the link (T_2).

Resolving the forces vertically,

$$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g \qquad \ldots(ii)$$

$$\ldots\left(\because T_2\cos\beta = \frac{M.g}{2}\right)$$

Resolving the forces horizontally,

$$T_1 \sin\alpha + T_2 \sin\beta = F_C$$

$$T_1 \sin\alpha + \frac{M.g}{2\cos\beta} \times \sin\beta = F_C \qquad \ldots\left(\because T_2 = \frac{M.g}{2\cos\beta}\right)$$

$$T_1 \sin \alpha + \frac{M.g}{2} \times \tan \beta = F_C$$

$\therefore$

$$T_1 \sin \alpha = F_C - \frac{M.g}{2} \times \tan \beta \quad \ldots (iii)$$

Dividing equation (*iii*) by equation (*ii*),

$$\frac{T_1 \sin \alpha}{T_1 \cos \alpha} = \frac{F_C - \frac{M.g}{2} \times \tan \beta}{\frac{M.g}{2} + m.g}$$

or

$$\left(\frac{M.g}{2} + m.g\right) \tan \alpha = F_C - \frac{M.g}{2} \times \tan \beta$$

$$\frac{M.g}{2} + m.g = \frac{F_C}{\tan \alpha} - \frac{M.g}{2} \times \frac{\tan \beta}{\tan \alpha}$$

Substituting $\frac{\tan \beta}{\tan \alpha} = q$, and $\tan \alpha = \frac{r}{h}$, we have

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q \quad \ldots (\because F_C = m.\omega^2.r)$$

or

$$m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$\therefore$

$$h = \left[m.g + \frac{M.g}{2}(1+q)\right]\frac{1}{m.\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2} \quad \ldots (iv)$$

or

$$\omega^2 = \left[m.g + \frac{Mg}{2}(1+q)\right]\frac{1}{m.h} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

or

$$\left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$\therefore$

$$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}\left(\frac{60}{2\pi}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h} \quad \ldots (v)$$

. . . (Taking $g = 9.81$ m/s^2)

Notes : 1. When the length of arms are equal to the length of links and the points P and D lie on the same vertical line, then

$$\tan \alpha = \tan \beta \text{ or } q = \tan \alpha / \tan \beta = 1$$

Therefore, the equation (*v*) becomes

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h} \quad \ldots (vi)$$

2. When the loaded sleeve moves up and down the spindle, the frictional force acts on it in a direction opposite to that of the motion of sleeve.

If F = Frictional force acting on the sleeve in newtons, then the equations (*v*) and (*vi*) may be written as

$$N^2 = \frac{m.g + \left(\frac{M.g \pm F}{2}\right)(1+q)}{m.g} \times \frac{895}{h} \quad \ldots (vii)$$

$$= \frac{m.g + (M.g \pm F)}{m.g} \times \frac{895}{h} \quad \ldots \text{(When } q = 1) \quad \ldots (viii)$$

The + sign is used when the sleeve moves upwards or the governor speed increases and negative sign is used when the sleeve moves downwards or the governor speed decreases.

3. On comparing the equation (*vi*) with equation (*ii*) of Watt's governor (Art. 18.5), we find that the mass of the central load (*M*) increases the height of governor in the ratio $\frac{m + M}{m}$.

2. *Instantaneous centre method*

In this method, equilibrium of the forces acting on the link *BD* are considered. The instantaneous centre *I* lies at the point of intersection of *PB* produced and a line through *D* perpendicular to the spindle axis, as shown in Fig. 18.4. Taking moments about the point *I*,

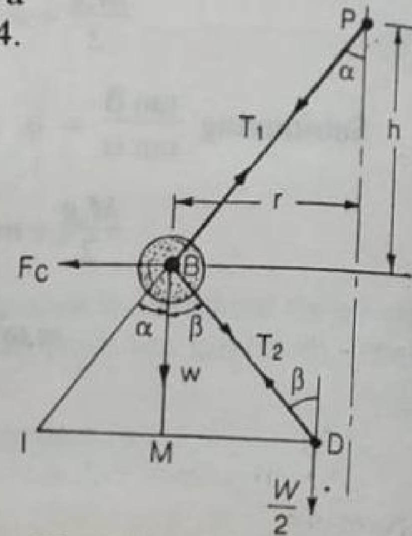


Fig. 18.4. Instantaneous centre method.

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID$$

$$= m.g \times IM + \frac{M.g}{2} \times ID$$

$$\therefore \quad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2} \times \frac{ID}{BM}$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right)$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM}{BM} + \frac{MD}{BM}\right)$$

$$= m.g.\tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta) \ldots \left(\because \frac{IM}{BM} = \tan\alpha, \text{ and } \frac{MD}{BM} = \tan\beta\right)$$

Dividing throughout by $\tan\alpha$,

$$\frac{F_C}{\tan\alpha} = m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right) = m.g + \frac{M.g}{2}(1+q) \quad \ldots \left(\because q = \frac{\tan\beta}{\tan\alpha}\right)$$

We know that $F_C = m.\omega^2.r$ and $\tan\alpha = \frac{r}{h}$

$$\therefore \quad m.\omega^2.r \times \frac{h}{r} = m.g + \frac{M.g}{2}(1+q)$$

or

$$h = \frac{m.g + \frac{M.g}{2}(1+q)}{m} \times \frac{1}{\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

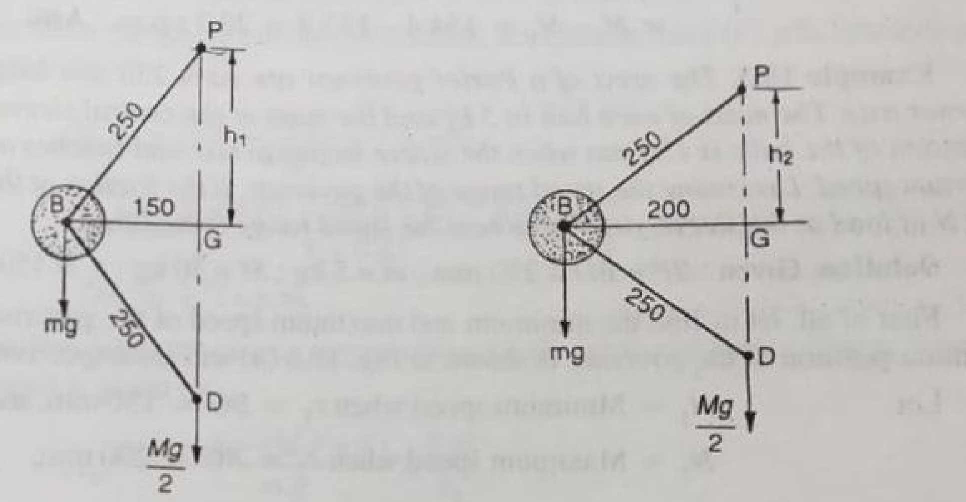
... (Same as before)

When $\tan\alpha = \tan\beta$ or $q = 1$, then

$$h = \frac{m + M}{m} \times \frac{g}{\omega^2}.$$

Example 18.2. *A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the minimum and maximum speeds and range of speed of the governor.*

Solution. Given : $BP = BD = 250$ mm $= 0.25$ m ; $m = 5$ kg ; $M = 15$ kg ; $r_1 = 150$ mm $= 0.15$m; $r_2 = 200$ mm $= 0.2$ m



(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.5

The minimum and maximum position of the governor is shown in Fig. 18.5 (*a*) and (*b*) respectively.

Minimum speed when $r_1 = BG = 0.15\ m$

Let N_1 = Minimum speed.

From Fig. 18.5 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.15)^2} = 0.2 \text{ m}$$

We know that

$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+15}{5} \times \frac{895}{0.2} = 17\,900$$

$\therefore$ $N_1 = 133.8$ r.p.m **Ans.**

Maximum speed when $r_2 = BG = 0.2\ m$

Let N_2 = Maximum speed.

From Fig. 18.5 (*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(0.25)^2 - (0.2)^2} = 0.15 \text{ m}$$

$$(N_2)^2 = \frac{m.g + \left(\frac{M.g + F}{2}\right)(1 + q_2)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{2 \times 9.81 + \left(\frac{15 \times 9.81 + 24}{2}\right)(1 + 0.703)}{2 \times 9.81} \times \frac{895}{0.1532} = 49\ 236$$

$\therefore \qquad N_2 = 222$ r.p.m.

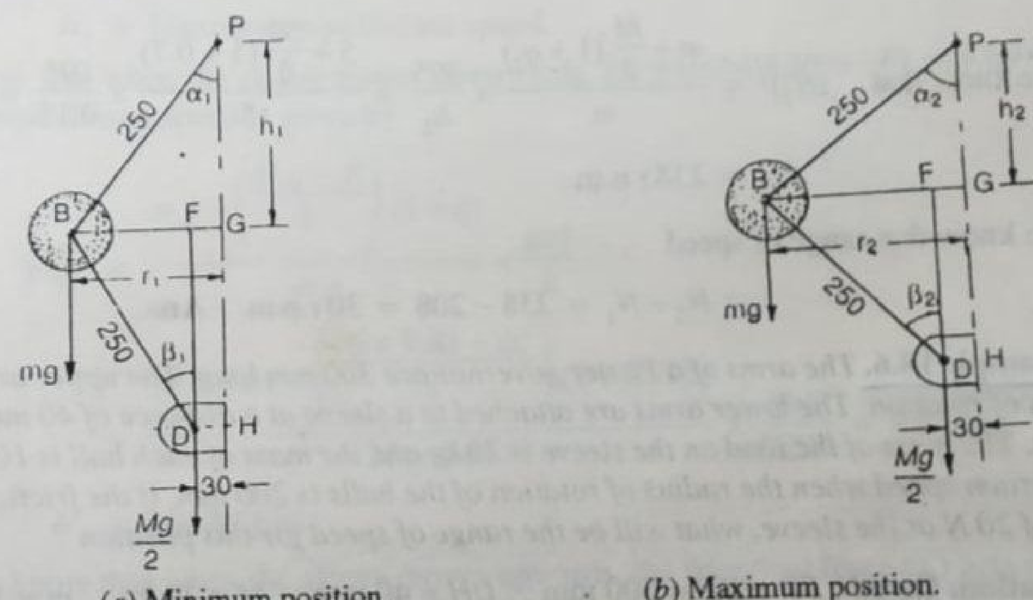
We know that range of speed

$$= N_2 - N_1 = 222 - 183.3 = 38.7 \text{ r.p.m.} \quad \textbf{Ans.}$$

Example 18.5. *A Porter governor has all four arms 250 mm long. The upper arms are attached on the axis of rotation and the lower arms are attached to the sleeve at a distance of 30 mm from the axis. The mass of each ball is 5 kg and the sleeve has a mass of 50 kg. The extreme radii of rotation are 150 mm and 200 mm. Determine the range of speed of the governor.*

Solution. Given : $BP = BD = 250$ mm ; $DH = 30$ mm ; $m = 5$ kg ; $M = 50$ kg ; $r_1 = 150$ mm ; $r_2 = 200$ mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. 18.8 (*a*) and (*b*) respectively.



(*a*) Minimum position. (*b*) Maximum position.

Fig. 18.8

Let $\qquad N_1$ = Minimum speed when $r_1 = BG = 150$ mm ; and

$\qquad N_2$ = Maximum speed when $r_2 = BG = 200$ mm.

From Fig. 18.8 (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

$$BF = BG - FG = 150 - 30 = 120 \text{ mm} \qquad \ldots (\because FG = DH)$$

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (120)^2} = 219 \text{ mm}$$

$\therefore$ $$\tan \alpha_1 = BG/PG = 150/200 = 0.75$$

and $$\tan \beta_1 = BF/DF = 120/219 = 0.548$$

$$\therefore \quad q_1 = \frac{\tan \beta_1}{\tan \alpha_1} = \frac{0.548}{0.75} = 0.731$$

We know that $$(N_1)^2 = \frac{m + \frac{M}{2}(1 + q_1)}{m} \times \frac{895}{h_1} = \frac{5 + \frac{50}{2}(1 + 0.731)}{5} \times \frac{895}{0.2} = 43\,206$$

$$\therefore \quad N_1 = 208 \text{ r.p.m.}$$

From Fig. 18.8 (*b*), we find that height of the governor,

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

$$BF = BG - FG = 200 - 30 = 170 \text{ mm}$$

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(250)^2 - (170)^2} = 183 \text{ mm}$$

$$\therefore \quad \tan \alpha_2 = BG/PG = 200 / 150 = 1.333$$

and $$\tan \beta_2 = BF/DF = 170 / 183 = 0.93$$

$$\therefore \quad q_2 = \frac{\tan \beta_2}{\tan \alpha_2} = \frac{0.93}{1.333} = 0.7$$

We know that $$(N_2)^2 = \frac{m + \frac{M}{2}(1 + q_2)}{m} \times \frac{895}{h_2} = \frac{5 + \frac{50}{2}(1 + 0.7)}{5} \times \frac{895}{0.15} = 56\,683$$

$$\therefore \quad N_2 = 238 \text{ r.p.m.}$$

We know that range of speed

$$= N_2 - N_1 = 238 - 208 = 30 \text{ r.p.m.} \quad \textbf{Ans.}$$

Example 18.6. *The arms of a Porter governor are 300 mm long. The upper arms are pivoted on the axis of rotation. The lower arms are attached to a sleeve at a distance of 40 mm from the axis of rotation. The mass of the load on the sleeve is 70 kg and the mass of each ball is 10 kg. Determine the equilibrium speed when the radius of rotation of the balls is 200 mm. If the friction is equivalent to a load of 20 N at the sleeve, what will be the range of speed for this position ?*

Solution. Given : $BP = BD = 300$ mm ; $DH = 40$ mm ; $M = 70$ kg ; $m = 10$ kg ; $r = BG = 200$ mm

Equilibrium speed when the radius of rotation $r = BG = 200$ mm

Let N = Equilibrium speed.

The equilibrium position of the governor is shown in Fig. 18.9. From the figure, we find that height of the governor,

$$h = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm}$$
$$= 0.224 \text{ m}$$

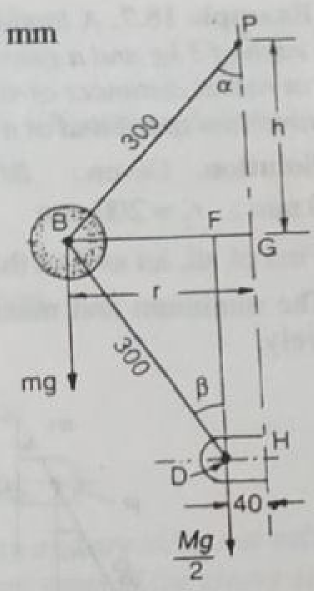
$$BF = BG - FG = 200 - 40 = 160 \text{ mm} \qquad \ldots (\because FG = DH)$$

and $\quad DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(300)^2 - (160)^2} = 254 \text{ mm}$

$\therefore \quad \tan \alpha = BG/PG = 200/224 = 0.893$

and $\quad \tan \beta = BF/DF = 160/254 = 0.63$

$$\therefore \quad q = \frac{\tan \beta}{\tan \alpha} = \frac{0.63}{0.893} = 0.705$$



All dimensions in mm

Fig. 18.9

We know that

$$N_2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

$$= \frac{10 + \frac{70}{2}(1+0.705)}{10} \times \frac{895}{0.224} = 27\,840$$

$\therefore \quad N = 167 \text{ r.p.m.}$ **Ans.**

Range of speed when friction is equivalent to load of 20 N at the sleeve (i.e. when F = 20 N)

Let $\quad N_1$ = Minimum equilibrium speed, and

$\quad N_2$ = Maximum equilibrium speed.

We know that when the sleeve moves downwards, the frictional force (F) acts upwards and the minimum equilibrium speed is given by

$$(N_1)^2 = \frac{m.g + \left(\frac{M.g - F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}$$

$$= \frac{10 \times 9.81 + \left(\frac{70 \times 9.81 - 20}{2}\right)(1+0.705)}{10 \times 9.81} \times \frac{895}{0.224} = 27\,144$$

$\therefore \quad N_1 = 164.8 \text{ r.p.m.}$

We also know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum equilibrium speed is given by

$$(N_2)^2 = \frac{m.g + \left(\frac{M.g + F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}$$

$$= \frac{10 \times 9.81 + \left(\frac{70 \times 9.81 + 20}{2}\right)(1+0.705)}{10 \times 9.81} \times \frac{895}{0.224} = 28\,533$$

$\therefore \quad N_2 = 169 \text{ r.p.m.}$

We know that range of speed

$$= N_2 - N_1 = 169 - 164.8 = 4.2 \text{ r.p.m.} \textbf{ Ans.}$$

18.8. Hartnell Governor

A Hartnell governor is a spring loaded governor as shown in Fig. 18.18. It consists of two bell crank levers pivoted at the points O,O to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm OB and a roller at the end of the horizontal arm OR. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.

Let m = Mass of each ball in kg,

M = Mass of sleeve in kg,

r_1 = Minimum radius of rotation in metres,

r_2 = Maximum radius of rotation in metres,

ω_1 = Angular speed of the governor at minimum radius in rad/s,

ω_2 = Angular speed of the governor at maximum radius in rad/s,

S_1 = Spring force exerted on the sleeve at ω_1 in newtons,

S_2 = Spring force exerted on the sleeve at ω_2 in newtons,

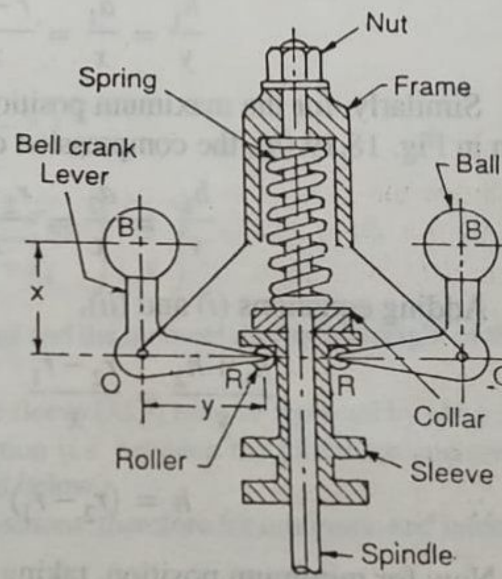


Fig. 18.18. Hartnell governor.

F_{C1} = Centrifugal force at ω_1 in newtons = $m\,(\omega_1)^2\,r_1$,

F_{C2} = Centrifugal force at ω_2 in newtons = $m\,(\omega_2)^2\,r_2$,

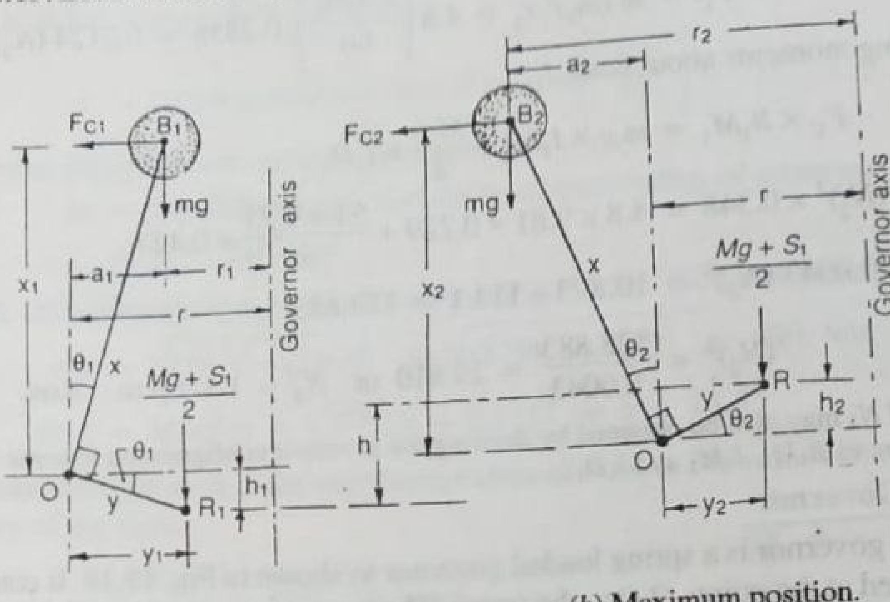
s = Stiffness of the spring or the force required to compress the spring by one mm,

x = Length of the vertical or ball arm of the lever in metres.

y = Length of the horizontal or sleeve arm of the lever in metres, and

r = Distance of fulcrum O from the governor axis or the radius of rotation when the governor is in mid-position, in metres.

Consider the forces acting at one bell crank lever. The minimum and maximum position is shown in Fig. 18.19. Let h be the compression of the spring when the radius of rotation changes from r_1 to r_2.



(a) Minimum position. (b) Maximum position.

Fig. 18.19

For the minimum position *i.e.* when the radius of rotation changes from r to r_1, as shown in Fig. 18.19 (*a*), the compression of the spring or the lift of sleeve h_1 is given by

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \qquad \ldots (i)$$

Similarly, for the maximum position *i.e.* when the radius of rotation changes from r to r_2, as shown in Fig. 18.19 (*b*), the compression of the spring or lift of sleeve h_2 is given by

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \qquad \ldots (ii)$$

Adding equations (*i*) and (*ii*),

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x} \qquad \ldots (\because h = h_1 + h_2)$$

$$\therefore \qquad h = (r_2 - r_1)\frac{y}{x} \qquad \ldots (iii)$$

Now for minimum position, taking moments about point O, we get

$$\frac{M.g + S_1}{2} \times y_1 = F_{C1} \times x_1 - m.g \times a_1$$

or

$$M.g + S_1 = \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1) \qquad \ldots (iv)$$

Again for maximum position, taking moments about point O, we get

$$\frac{M.g + S_2}{2} \times y_2 = F_{C2} \times x_2 + m.g \times a_2$$

or

$$M.g + S_2 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) \qquad \ldots (v)$$

Subtracting equation (*iv*) from equation (*v*),

$$S_2 - S_1 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) - \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1)$$

We know that

$$S_2 - S_1 = h.s, \text{ and } h = (r_2 - r_1)\frac{y}{x}$$

$$\therefore \quad s = \frac{S_2 - S_1}{h} = \left(\frac{S_2 - S_1}{r_2 - r_1}\right)\frac{x}{y}$$

Neglecting the obliquity effect of the arms (*i.e.* $x_1 = x_2 = x$, and $y_1 = y_2 = y$) and the moment due to weight of the balls (*i.e.* $m.g$), we have for minimum position,

$$\frac{M.g + S_1}{2} \times y = F_{C1} \times x \quad \text{or} \quad M.g + S_1 = 2F_{C1} \times \frac{x}{y} \qquad \ldots (vi)$$

Similarly for maximum position,

$$\frac{M.g + S_2}{2} \times y = F_{C2} \times x \quad \text{or} \quad M.g + S_2 = 2F_{C2} \times \frac{x}{y} \qquad \ldots (vii)$$

Subtracting equation (*vi*) from equation (*vii*),

$$S_2 - S_1 = 2(F_{C2} - F_{C1})\frac{x}{y} \qquad \ldots (viii)$$

We know that

$$S_2 - S_1 = h.s, \text{ and } h = (r_2 - r_1)\frac{y}{x}$$

$$\therefore \quad s = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{C2} - F_{C1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2 \qquad \ldots (ix)$$

Notes : 1. Unless otherwise stated, the obliquity effect of the arms and the moment due to the weight of the balls is neglected, in actual practice.

2. When friction is taken into account, the weight of the sleeve ($M.g$) may be replaced by ($M.g \pm F$).

3. The centrifugal force (F_C) for any intermediate position (*i.e.* between the minimum and maximum position) at a radius of rotation (r) may be obtained as discussed below :

Since the stiffness for a given spring is constant for all positions, therefore for minimum and intermediate position,

$$s = 2\left(\frac{F_C - F_{C1}}{r - r_1}\right)\left(\frac{x}{y}\right)^2 \qquad \ldots (x)$$

and for intermediate and maximum position,

$$s = 2\left(\frac{F_{C2} - F_C}{r_2 - r}\right)\left(\frac{x}{y}\right)^2 \qquad \ldots (xi)$$

$\therefore$ From equations (*ix*), (*x*) and (*xi*),

$$\frac{F_{C2} - F_{C1}}{r_2 - r_1} = \frac{F_C - F_{C1}}{r - r_1} = \frac{F_{C2} - F_C}{r_2 - r}$$

or

$$F_C = F_{C1} + (F_{C2} - F_{C1})\left(\frac{r - r_1}{r_2 - r_1}\right) = F_{C2} - (F_{C2} - F_{C1})\left(\frac{r_2 - r}{r_2 - r_1}\right)$$