

Design of Machine Elements DME/S6

Design of cast Iron (C.I.) Pulley

Prob: Design a C.I. Pulley (Driver) which transmit a Power of 15KW. Speed of the motor shaft 500 rev./min. Speed ratio is 4 and there is considerable variation of speed. For material for shaft allowable shear stress is 50 N/mm^2 , for key material, allowable shear stress = 40 N/mm^2 allowable crushing stress = 100 N/mm^2 for cast iron, allowable tensile stress is 20 N/mm^2 and allowable shear stress is 15 N/mm^2 . Assume any data if reqd.

Solⁿ

$$P = 15 \text{ KW.}$$

$$N = 500 \text{ rev/min} \quad \text{Speed ratio} = 4$$

For shaft. Key material

$$\tau_s = 50 \text{ N/mm}^2$$

$$\tau_s = 40 \text{ N/mm}^2$$

$$\sigma_c = 100 \text{ N/mm}^2$$

For C.I.

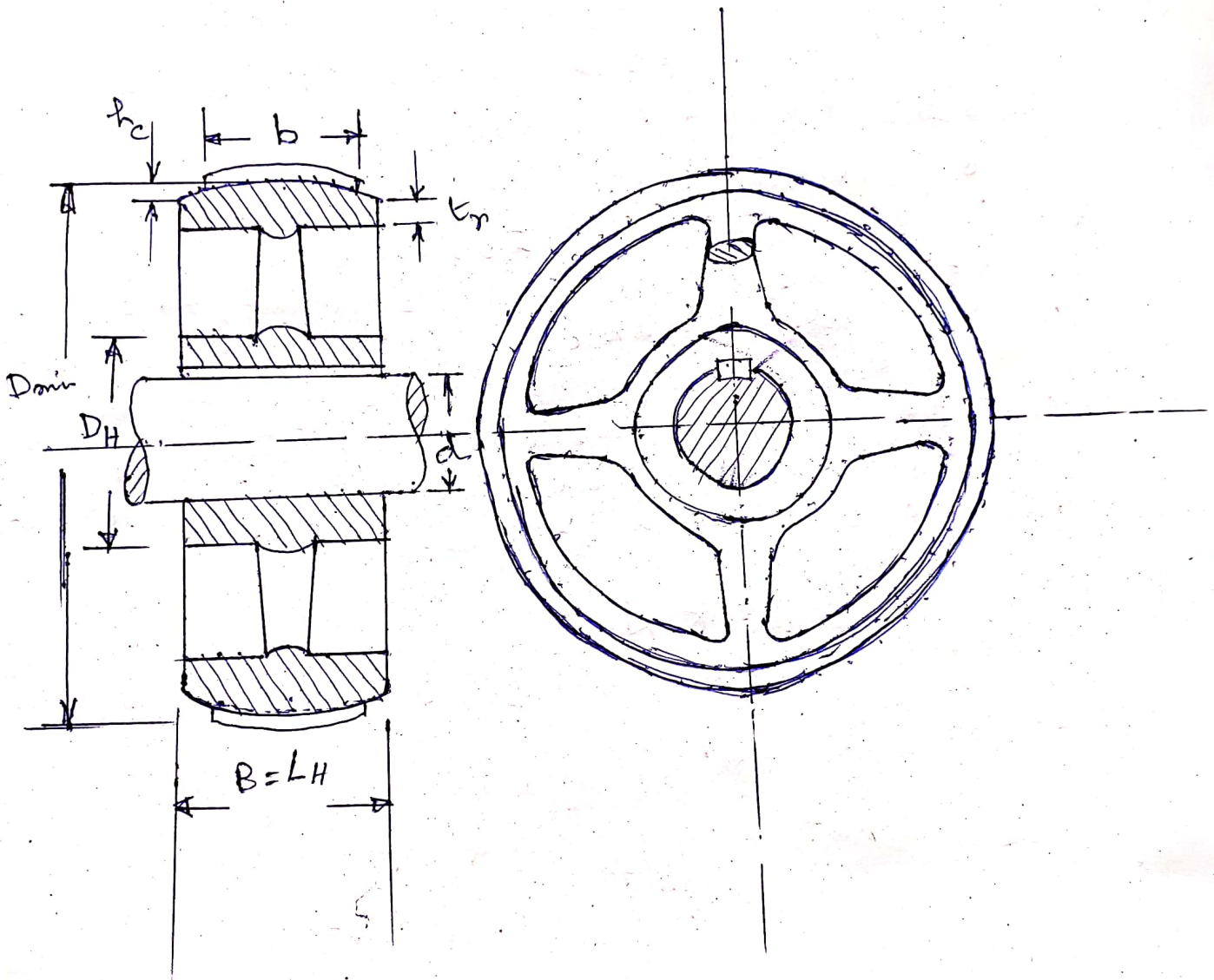
$$\sigma_t = 20 \text{ N/mm}^2$$

$$\tau_s = 15 \text{ N/mm}^2$$

Taking service factor is 1.3



(1A)





(2)

Dia. of the Pulley

The dia. of the motor (driver) shaft pulley can be found by the relation

$$D_{\min} = 1.10 \text{ to } 1.33 \sqrt[3]{\frac{P}{N}}$$

where, P is the power in kW and N is the r.p.m. of the shaft.

$$\text{We take } D_{\min} = 1.3 \sqrt[3]{\frac{P}{N_{\max}}}$$

Since there is considerable variation of load, so let us take the service factor as 1.3. Therefore the design power

$$P = \text{Power to be transmitted} \times \text{Service factor} \\ = 15 \times 1.3 = 19.5 \text{ kW}$$

$$N_{\max} = 500 \text{ rev./min}$$

$$\text{So, } D_{\min} = 1.3 \sqrt[3]{\frac{19.5}{500}} = 0.4408 \text{ m}$$

Say 0.45 m. or. 45 cm.

So, the dia. of the driven pulley is 180 cm = 1.80 m. neglecting slip between belt and the pulley

Belt speed is V ,

$$\text{So, } V = \frac{\pi D N}{60} = \frac{\pi \times 0.45 \times 500}{60} = 11.775 \text{ m/sec.}$$

Since the belt speed is more than 10 m/sec. therefore the centrifugal force will be considered.



(3)

The angle of contact between the belt and the smaller pulley is θ .

$$\text{So, } \theta = 180 - 2 \sin^{-1} \frac{D_1 - D}{2c}$$

where, D_1 is the dia. of the larger (driver) pulley

c is the centre distance between pulley

then $c \approx 3.5 D_1$ i.e. 6.3 m .

$$\theta = 180 - 2 \sin^{-1} \frac{1.8 - 0.45}{2 \times 6.3} = 167.7^\circ = 2.92 \text{ rad.}$$

now, Power, $P = \frac{(T_1 - T_2) \times v}{1000}$ where, T_1 & T_2 are the tension on tight side & tension on slack side of the belt respectively.

$$19.5 = \frac{(T_1 - T_2) \times 11.775}{1000}$$

$$\therefore T_1 - T_2 = \frac{19.5 \times 1000}{11.775} = 1656.05 \text{ N. } \textcircled{1}$$

$$\text{now, } \frac{T_1}{T_2} = e^{\mu \theta}$$

where, μ = co-efficient of friction between leather belt & pulley & may be taken as 0.35

$$\text{So, } \frac{T_1}{T_2} = e^{0.35 \times 2.92} = 2.77 \text{ --- } \textcircled{2}$$

From Equ. $\textcircled{1}$ & $\textcircled{2}$

$$T_1 = 2591.67 \text{ N} \quad T_2 = 935.62 \text{ N}$$



(7)

We know, the centrifugal tension of a belt is given by $T_c = m U^2$ Newton.

where, $m = \rho l b t$

m = mass of belt in kg per meter length of belt

ρ = density of belt material = 1000 kg/m^3 for leather belt.

l = length of belt = 1 m .

b = width of belt, in m

t = thickness of belt in m

So, taking thickness of belt $t = 0.03 \text{ m}$
 $\approx 3.5 \text{ mm}$ say 4 mm .

$$\text{So, } T_c = 1000 \times 1 \times b \times 0.016 \times (11.775)^2 \\ = 1941.10 b \text{ Newton}$$

Now, the working stress of leather belt is varying from 2.1 MPa to 3.15 MPa .

So we take, 3.0 MPa is the working stress of leather belt.

$$\text{Now, } \sigma_{\text{max}} = \frac{T_1 + T_c}{b \times t}$$

$$\therefore 3 \times 10^6 = \frac{2591.67 + 1941.10b}{b \times 0.016}$$

$$\therefore 42000b = 2591.67 + 1941.10b$$

$$\therefore b = 0.06469 \text{ m} = 64.69 \text{ mm} \text{ say } 65 \text{ mm}$$

width of the pulley face, $B = 1.25b = 1.25 \times 65$
 $= 81.25 \text{ mm}$.

say 82 mm .

(5)

According to empirical formula, the rim thickness of the pulley, t_r

$$\begin{aligned} \text{So, } t_r &= \frac{D_{\text{min}}}{300} + 2 \text{ mm.} \\ &= \frac{450}{300} + 2 = 3.5 \text{ mm.} \end{aligned}$$

Another relation of the rim thickness

$$\begin{aligned} t_r &= (0.005 D_{\text{min}} + 0.003) \text{ m} \\ &= (0.005 \times 0.45 + 0.003) = 0.00525 \text{ m.} \\ &= 5.25 \text{ mm.} \end{aligned}$$

We take $t_r = 6 \text{ mm}$.

Crown height = 1.5 mm per metre of face width.
is about 1.5 mm. Taken, $h_c = 1.5 \text{ mm}$.

Design of hub

d = dia. of the shaft = inner dia. of the hub

D_H = outer dia. of the hub.

L_H = length of the hub

P = Power transmitted by the shaft
taken with service factor
= 19.5 kW

$$\text{So, } P = \frac{2\pi T_{\text{max}} \cdot N}{60 \times 1000}$$

$$19.5 = \frac{2\pi \times 500 \times T_{\text{max}}}{60 \times 1000}$$

$$\therefore T_{\text{max}} = 372.61 \text{ N-m.}$$





(6)

Equivalent torque on the shaft is given by

$$T_E = \sqrt{M^2 + T_{\max}^2}$$

where, M = Maximum bending moment on the shaft is taken as zero, since weight of the pulley is to be neglected

$$\therefore T_E = T_{\max} = \frac{\pi d^3 \tau_s}{16}$$

$$\therefore 372.61 \times 10^3 = \frac{\pi d^3 \times 50}{16}$$

$$\therefore d = 33.60 \text{ mm. Say } 34 \text{ mm.}$$

According to empirical relation, hub dia D_H

$$D_H = 1.75d + 7 \text{ mm.}$$

$$= 1.75 \times 34 + 7 = 66.5 \text{ mm.}$$

Say 68 mm.

As other empirical relation is $D_H = 2d$

L_H = length of the hub

$$= \frac{2}{3} B \text{ to } B$$

$$L_H = B = 82 \text{ mm.}$$

Checking for shear stress of the hub material which is made of cast iron.

$$T_{\max} = \frac{\pi}{16} \frac{(D_H^4 - d^4)}{D_H} \tau_{s \text{ cf.}}$$

$$372.61 \times 10^3 = \frac{\pi}{16} \frac{\{(68)^4 - (34)^4\}}{68} \times \tau_{s \text{ cf.}}$$

$$= 6.49 \text{ N/mm}^2$$



(7)
which is less than the allowable shear stress
 15 N/mm^2 . Hence safe

Design of key

$$W_k = \text{width of key} = \frac{d}{4} = \frac{34}{4} = 8.5 \text{ mm say } 9 \text{ mm.}$$

$$d_k = \text{depth of key} = \frac{d}{6} = \frac{34}{6} = 5.66 \text{ mm say } 6 \text{ mm.}$$

Shear failure of key

$$T_{\text{max}} = W_k \times l_k \times \tau_{sk} \times \frac{d}{2}$$

where, l_k = length of key

τ_{sk} = Allowable shear stress of
key material = 40 N/mm^2

So, according to relation

$$372.61 \times 10^3 = 9 \times l_k \times 40 \times \frac{34}{2}$$

$$\Rightarrow l_k = 60.88 \text{ mm. say } 61 \text{ mm.}$$

Crushing failure of the key

$$T_{\text{max}} = l_k \times \frac{d_k}{2} \times \sigma_{ck} \times \frac{d}{2}$$

where, σ_{ck} = allowable crushing stress of
the key material = 100 N/mm^2

So, according to relation

$$372.61 \times 10^3 = l_k \times \frac{6}{2} \times 100 \times \frac{34}{2}$$

$$\text{or, } l_k = 73.06 \text{ mm.}$$

So, we take $l_k = 74 \text{ mm.}$

So, actual length of is equated l_k plus 12 mm.

for quick fitting & dismantling

$$\text{So, } l_{kac} = 74 + 12 = 86 \text{ mm.}$$



(8)

Design of arms.

Since the dia. of the Pulley is 45 cm, we should take the no. of arms is 4 and the section of the arm are elliptical.

Let, a_1 = length of the major axis near the hub.

b_1 = length of the minor axis near the hub

Then, we know that the maximum bending stress set up in the arm material (C.I.) is given by

$$\frac{M}{I} = \frac{\sigma}{y}$$

where, M = Maximum Bending moment per arm

$$n = \text{no. of arm} = 4 \quad = \frac{T_{max}}{n/2}$$

$$\frac{n}{2} = \text{no. of effective arm.}$$

I = Moment of Inertia of the ellipse at hub end about neutral axis.

$$= \frac{\pi}{64} b_1 a_1^3$$

length of minor axis b_1 is half of the length of major axis a_1

$$\therefore b_1 = \frac{a_1}{2}$$

σ = maximum bending stress set up in the Pulley.

$$\therefore \frac{T_{max}}{n/2} = \frac{\sigma}{\frac{a_1}{2}} \quad \therefore \sigma = \frac{2 T_{max}}{\frac{\pi}{64} b_1 a_1^3} \times \frac{a_1}{2}$$



$$\tau = \frac{128 T_{max}}{\pi \tau a_1^3} \quad (9)$$

$$a = 20 \text{ mm}^2$$

$$\therefore 20 = \frac{128 \times 372.61 \times 10^3}{4 \times \pi \times a_1^3}$$

$$\therefore a_1 = 57.46 \text{ mm. say } 58 \text{ mm.}$$

$$b_1 = \frac{58}{2} = 29 \text{ mm.}$$

Taper of the arms = 1:25

Length of each arm is L_{arm} .

$$\begin{aligned} \therefore L_{arm} &= \frac{(D_{min} - 2 \times \text{crow's height} - 2 \times \text{rim thickness} - D_H)}{2} \\ &= \frac{(450 - 2 \times 1.5 - 2 \times 6 - 68)}{2} \\ &= 183.5 \text{ mm.} \end{aligned}$$

So, a_2 = length of major axis at rim end.

b_2 = length of minor axis at rim end.

$$a_2 = 43 \text{ mm.} \quad b_2 = 22 \text{ mm.}$$

Final Data

$$D_{min} = 450 \text{ mm.}$$

$$b = 65 \text{ mm.}$$

$$B = 82 \text{ mm.}$$

$$d = 34 \text{ mm.}$$

$$D_H = 68 \text{ mm.}$$

$$L_H = 82 \text{ mm.}$$

$$t_r = 6 \text{ mm}$$

$$h_c = 1.5 \text{ mm.}$$

$$w_r = 9 \text{ mm.}$$

$$d_r = 6 \text{ mm}$$

$$L_r = 86 \text{ mm.}$$

$$a_1 = 58 \text{ mm.}$$

$$b_1 = 29 \text{ mm.}$$

$$a_2 = 43 \text{ mm.}$$

$$b_2 = 22 \text{ mm.}$$

Taper of arm 1:25