Mr. Arup Kumar Majee,Principal
Jnan Chandra Ghosh Polytechnic

Chapter 1

Basic concepts & Laws

2.1 Basic concept of charge, current, voltage, resistance,inductance, Capacitance, power, energy and their units.
2.2 Basic concept about supply source- D.C. & A.C. (names only)
3.1 Statement & explanation of (a) Ohm's law, resistances in series and parallel (b) Kirchhoff's Current & Voltage laws
3.2 Simple problems on D.C. Circuits

Learning outcomes:

On completion of this chapter, learners should be able to

- classify materials on the basis of electrical conductivity.
- define different terminologies often used in Electrical Engineering.
- state and explain Ohm's law.
- define resistance and conductance.
- calculate equivalent resistance when resistances are connected in series or parallel.
- convert star connection to delta or delta connection to star.
- define ideal and practical inductance and quality factor.
- calculate equivalent inductance when inductances (not mutually coupled) are connected in series or parallel.
- define ideal and practical capacitance and quality factor.
- calculate equivalent capacitance when capacitances are connected in series or parallel.
- state and explain Kirchhoff's laws and solve network problems using these laws.
- define and classify energy sources.
- convert a voltage source to a current source and vice versa.

1.1 Classification of materials

On the basis of electrical conductivity, materials may be classified into following categories.

Conductors: Materials like copper, aluminum etc. valence band and conduction band are overlapped. Even a little potential difference may cause a large flow of current (caused by the movement of electrons). These materials are called conductors.

Insulators: In materials like wood, mica etc. there is a very large forbidden energy gap (nearly 15eV) between valence band and conduction band. Hence, electrons cannot be easily detached from the atoms unless a very high electric field is applied. Under normal conditions they do not allow any passage of current. These materials are called insulators.

Semiconductors: In materials like germalium, silicon etc. forbidden gap between valence band and conduction band is small (1eV). Even under room temperature some electrons are available in conduction band to take part in the flow of current. If temperature is increased more electrons are available. Even though they behave like insulators at a lower temperature, they behave like a conductor at higher temperature. Their conductivity lies between insulators and conductors. That is why they are called semiconductors.

1.2 Terminologies used

Charge: It is the attribute of matter which is responsible for a force of interaction between two matters under certain conditions. It is considered as the quantity of electricity. Charge is measured in coulomb. It is defined as an amount of charge possessed by 6.242×10^{18} electrons.

Electric Current: An electric current is caused by the movement of electric charge around a closed path. It is defined as the rate of flow of positive charge.

$$i = \frac{dq}{dt}$$

Unit of Current, Ampere: Current is measured in amperes. One Ampere current means flow of 1C charge passing any point of conductor in 1sec.
The ampere may also be defined as the current which when flowing through two infinitely long conductors having negligibly small cross-sectional area, situated 1 metre apart in the vacuum, produces a force of $2 \times 10^{-7} N/metre$ between the conductors.

Example 1.1 *If 6 coulomb charge passes through copper conductor in 1 minute, what is the value of current?*

Solution Given values

$$Q = 6\ coulomb$$
$$t = 1min = 60sec$$
$$\therefore I = \frac{Q}{t} = \frac{6}{60} = 0.1A$$

Current flowing through the condutor is 0.1A.

Electric current density: It is defined as the current per unit area of the conducting medium. Considering uniform current distribution

$$J = \frac{I}{A}$$

where A is the cross-sectional area of the conductor. J is measured in ampere per square metre.

Electric Potential: The electric potential V of a point A with respect to another point B is the work done against the field in taking a positive charge from point B to point A.

$$v = \frac{dW}{dq}$$

Electromotive force: It is the voltage provided by an energy source which forces a current to flow in a closed circuit.

Voltage drop: Voltage drop between two points is defined as the decrease in electric potential energy in transferring a unit positive charge from one point to the other.

Unit of voltage, Volt: E.m.f, potential, voltage drop all are measured by volts. One volt is the measure of voltage between two points when 1J of energy is required to transfer 1 coulomb of charge from one point to another.

Power: It is defined as the rate at which the work is being done in an electric circuit. It may also be defined as the rate at which energy is being consumed in an electric circuit.
If in time Δt, a small charge Δq is moved through an element having a voltage v across it,

$$P = \lim_{t \to 0} \frac{\Delta W}{\Delta t} = \frac{dw}{dt} = v\frac{dq}{dt} = vi$$

Electric power is measured in Watts.

Electric energy: The energy which is caused by the movement of electrons from one place to another is called **electric energy**. It is measured in Joules.

If in time Δt, a small charge Δq flows through an element having a voltage v across it,

$$W = \int dw = \int vdq = \int vidt = \int pdt$$

Electric energy may be defined as total power consumed over a specific time. It can be measured in Watt-sec (practical unit used: kilowatt-hours).

Electrical Networks and circuits: Any combination of electrical elements is known as a network . A network may contain active elements or not. But circuit is a closed energized network. A circuit must contain an active element.

Therefore, ***all circuits are networks but all networks are not circuits***.

Nodes and branches A node is a point where two or more circuit elements join. If three or more elements are joined at a node the node is called an essential node.

A branch is a path which connects two nodes.

Loops and meshes: A loop or mesh is a closed path obtained by starting at a node and returning at the same node through a set of connected circuit elements without passing any intermediate node more than once.

A mesh is the smallest loop. it does not contain any other loop.

Therefore, ***all meshes are loops but all loops are not meshes***.

Circuit element: A circuit element is any two terminal component which constitutes a circuit. Energy sources, diodes, resistors, inductors etc. are circuit elements.

Active element and passive elements: An active element can generate electric energy. Any voltage source or current source is an active element.

A passive element cannot generate electric energy but it can either consume or store it. Resistors, inductors and capacitors are commonly used passive elements. A resistor dissipates energy whereas an inductor stores energy in magnetic field and a capacitor stores energy in electrostatic field.

Bilateral and unilateral elements: For a bilateral element , the same relationship between current and volatge exists for either direction of current flow. Resistors, inductors are bilateral elements.

On the other hand, different current voltage relationships obtained for both direction of current flow for unilateral elements. Diodes, transistors etc. are unilateral elements.

1.3 Electrical Circuit Elements

There are four basic circuit elements: resistors, inductors, capacitors and energy sources.

1.3.1 Resistor

Ohm's law

Statement: Physical conditions (tempertaure, material etc.) remaining constant, current (I) through any conductor is directly proportional to the potential difference (V) between its ends.

$$I \propto V$$

Or, $I = G.V$ where proportionality constant, G is the conductance measured in mho.
Or, $V = R.I$ where proportionality constant, $R = 1/G$ is the resistance measured in Ohm.

In other words, Ohm's law may be stated as: potential difference between the two ends of a conductor is directly proportional to the current flowing through it provided physical conditions remain constant.

Example 1.2 *The potential drop across* 15Ω *resistor is 60V. What will be the value of current flowing through it?*

Figure 1.1: Symbol of a resistor

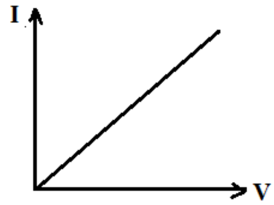


Figure 1.2: Voltage-ampere characteristics of a linear resistor

Solution Given values

Resistance, $R = 15\Omega$

Potential drop, $V = 60V$

$\therefore$Current flowing through the resistor, $I = \frac{V}{R} = \frac{60}{15} = 4A$

Resistance, R Resistance is the property of a material which resists the flow of current through it. It is measured in Ohms (Ω).

$$R = \frac{V}{I}$$

A conductor has a resistance of 1Ω while carrying 1A current it produces a voltage drop of 1V across it.

Resistor: A two terminal electrical device (shown in Figure 1.1) which has resistance as its dominant characteristic is called a **resistor**. It is a circuit element which consumes power and dissipates energy as heat while carrying current. A resistor which follows Ohm's law is called a linear resistor and which does not follow Ohm's law is called a non-linear resistor.

Power consumed by a resistor having resistance $R\,\Omega$, carrying a direct current, I

$$P = VI = I^2R = \frac{V^2}{R}(\text{in watts})$$

Energy dissipated in a resistor over a time duration T sec,

$$W_d = PT = VIT = I^2RT(\text{in joules})$$

While carrying an alternating current, i instantaneous power absorbed by a resistor $p = vi$ and energy dissipated

$$W_d = \int_0^T vidt = \int_0^T Ri.idt = \int_0^T i^2Rdt = I_{rms}^2RT$$

where, I_{rms} is the root mean square value of an alternating current.

Example 1.3 *What is the value of current flowing through a 1500W electric iron when connected to a 230V, 50 Hz ac source ?*

Solution Given values

Power rating of heater, $P = 1500W$

Supply voltage, $V = 230V$

Current flowing through the heater, $I = \frac{P}{V} = \frac{1500}{230} = 6.5217A$

DC resistance of a conductor : While direct current flows through a conductor, current flows uniformly through the conductor. Resistance may be expressed as

$$R = \rho\frac{l}{A}$$

where l is the length of a conductor measured in metres.

A is the cross-sectional area of a conductor measured in square metres.

ρ is the resistivity of the conductor material measured in ohm-metre.

Resistivity: Resistivity is the ability of a material to oppose the flow of current.

$$[\rho] = \frac{[R][A]}{[L]} = \frac{\Omega \times m^2}{m} = \Omega - m$$

Resistivity is measured in $\Omega - m$

Conductance: Conductance, G is the reciprocal of resistance. It is measured in mho (℧) or siemens (S).

$$G = \frac{1}{R} = \sigma \frac{A}{l}$$

where σ is the conductivity of the conductor material measured in mho per metre.

Conductivity Conductivity is the ability of the material to facilitate the flow of current.

$$[\sigma] = \frac{[G][L]}{[A]} = \frac{\mho \times m}{m^2} = \mho/m$$

Conductivity is measured in $\mho/m$

AC Resistance of a conductor: While alternating current flows through a conductor, current cannot flow uniformly through the conductor. Due to reactive effect current shows a tendency to flow through the surface of the conductor which is known as *skin effect*. Effective cross sectional area of conductor reduces due to this, thus increasing the resistance.

Usually ac resistance is considered 1.2 to 1.3 times the dc resistance of a conductor.

Example 1.4 *The resistance of a heating element made of nichrome wire is* 25Ω. *The diameter of the wire is 0.235 mm and the resistivity is* $1.2\mu\Omega - m$. *What is the length of the wire used?*

Solution Let R be the resistance of the nichrome wire having resistivity, length and cross-sectional area ρ, l and A respectively.

Given values

$$\begin{aligned}
\rho &= 1.2\ \mu\Omega - m \\
d &= 0.235\ mm = 0.235 \times 10^{-3}\ m \\
A &= \pi \frac{d^2}{4} = \pi \frac{\left(0.235 \times 10^{-3}\right)^2}{4}\ m^2 \\
l &= \frac{R.A}{\rho} = \frac{25 \times \pi \frac{\left(0.235 \times 10^{-3}\right)^2}{4}}{1.2 \times 10^{-6}} = 0.9036\ m
\end{aligned}$$

Example 1.5 *In the manufacturing process of a copper wire, a thick circular rod which has a resistance of* 0.01Ω, *is drawn out without a change in volume until its diameter is one-tenth of what it was. What is the new resistance?*

Solution Let R_1 be the resistance of the copper rod having volume, length and cross-sectional area V_1, l_1 and A_1 respectively.

Let R_2 be the resistance of the copper wire having volume, length and cross-sectional area V_2, l_2 and A_2 respectively.

$$\begin{aligned}
d_2 &= \frac{d_1}{10} \\
V_1 &= V_2 \\
\text{Or, } l_1 A_1 &= l_2 A_2 \\
\text{Or, } \frac{l_1}{l_2} &= \frac{A_2}{A_1} = \left(\frac{d_2}{d_1}\right)^2 = \frac{1}{100} \\
R_1 &= \rho \frac{l_1}{A_1} \\
R_2 &= \rho \frac{l_2}{A_2} \\
\therefore \frac{R_1}{R_2} &= \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \frac{1}{10000} \\
R_2 &= 10000 \times 0.01 = 100\ \Omega
\end{aligned}$$

Example 1.6 *The A current of 2.5A flows in a coil of copper wire having cross sectional area of* $0.17mm^2$ *when connected to a 50V dc supply. Find the length of the wire on the coil if the resistivity of copper is* $17\mu\Omega - mm$.

Solution Let R be the resistance of the copper wire having resistivity, length and cross-sectional area ρ, l and A respectively.

Given values

$$\begin{aligned} V &= 50\ volt \\ I &= 2.5\ A \\ \rho &= 17\ \mu\Omega - mm = 17 \times 10^{-6} \times 10^{-3}\ \Omega - m \\ R &= \frac{V}{I} = \frac{50}{2.5} = 20\ \Omega \\ l &= \frac{R.A}{\rho} = \frac{20 \times 0.17 \times 10^{-6}}{17 \times 10^{-6} \times 10^{-3}} = 200\ m \end{aligned}$$

1.3.2 Inductor

Inductance, L When an electric current flows through a circuit it produces a flux around it. Inductance or self inductance is defined as the ratio of change in magnetic flux linkage to the change in current.

$$L = N\frac{d\phi}{di}$$

It is measured in henry (H). 1H=1wb/A

(a) air-core (b) iron-core

Figure 1.3: Symbol of an inductor

Inductance may be defined as the attribute of an electrical device which opposes instantaneous change in current. This may be compared with the property of inertia.

In direct current circuits, its effect is felt during switching on or switching off. As there is no change in flux during steady state, it acts like a short circuit. But in alternating current circuits, its presence is always there.

Inductor: It is a two terminal electrical device which can store energy in magnetic field . It is usually made by winding a coil (shown in Figure 1.3) on a magnetic core (for smaller values, air-core is used). For larger currents core becomes saturated and inductor value reduces, thus non-linearity is introduced.

$$\text{Energy stored} : W_s = \int vidt = \int L\frac{di}{dt}idt = \int Lidi = \frac{1}{2}Li^2$$

Example 1.7 *What will be the induced emf in a coil if current flowing through it reduces from 10A to 5A in 0.1sec? Assume self inductance of the coil is 5H.*

Solution Induced emf,

$$e = -L\frac{di}{dt} = -L\frac{I_2 - I_1}{dt} = -5 \times \frac{(5-10)}{0.1} = 250V$$

Example 1.8 *How much energy is stored in an inductor of 40 mH when a current of 4 A is passing through it?*

Solution Energy stored,

$$E = \frac{1}{2}Li^2 = \frac{1}{2} \times 40 \times 10^{-3} \times 4^2 = 0.32J$$

$$i = \frac{dq}{dt} = C\frac{dv_c}{dt}$$
$$\text{Or, } v_c = \frac{1}{C}\int idt$$

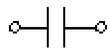


Figure 1.4: Symbol of a capacitor

1.3.3 Capacitor

Capacitance is the measure of change in electric charge stored for a certain change in potential.

$$C = \frac{dq}{dv_c}$$

Capacitance is measured in Farad (F). 1 Farad=1 coulomb per volt.

Capacitance may be defined as the attribute of an electrical device which opposes instantaneous change in voltage across it. In direct current circuits, its effect is felt during switching on or switching off. As there is no change in charge during steady state, it acts like an open circuit. But in alternating current circuits, its presence is always there.

Capacitor: It is a two terminal electrical device (shown in Figure 1.4) which can store energy in electrostatic field. It is made of two metal plates separated by some dielectric.

$$C = \epsilon\frac{A}{d} = \epsilon_0\epsilon_r\frac{A}{d}$$

where C is the capacitance in Farad
ϵ is the absolute permittivity of insulator
ϵ_o is the permittivity of vacuum
ϵ_r is the relative permittivity of insulator
A is the area of each plate in sq, metre
d is the separation between the plates in metre.

Example 1.9 *A capacitor comprises of two square metal plates each of side 120mm separated by a dielectric of thickness 2mm and permittivity 5. Determine the value of the capacitance.*

Solution

$$C = \frac{\epsilon_o\epsilon A}{d} = \frac{1}{36\pi \times 10^9} \times \frac{5 \times 120^2 \times 10^{-6}}{2 \times 10^{-3}} = 318.31pF$$

Example 1.10 *The dielectric thickness between the plates of a capacitor is 2mm and its relative permittivity is 5. The area of the plates is 50 square millimeters. The dielectric medium has electric field strength of 400V per mm. Find (a) capacitance; (ii) total charge on each plate.*

Solution

$$C = \frac{\epsilon_o\epsilon A}{d} = \frac{1}{36\pi \times 10^9} \times \frac{5 \times 50^2 \times 10^{-6}}{2 \times 10^{-3}} = 1.1052\mu F$$
$$V = 400 \times 2 = 800V$$
$$Q = CV = 1.1052 \times 10^{-6} \times 800 = 884.19\mu C$$

1.4 Connection of Circuit Elements

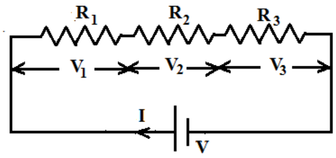
Circuit elements are connected in various ways to form networks. Commonly used connections are: series, parallel, star and delta.

1.4.1 Series Connection of Resistors

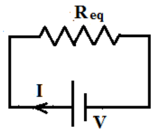
Resistors are said to be connected in series when same current flows through them. Figure 1.5a shows three resistors R_1, R_2 and R_3 connected in series. Potential drops are V_1, V_2 and V_3 across the each resistor respectively. Supply voltage V must be equal to the sum of these three voltage drops. Figure 1.5b shows a resistor R_{eq} which draws the same current when connected to the same source.

Applying Kirchhoff's voltage law:

$$V = V_1 + V_2 + V_3$$
$$\text{Or, } IR_{eq} = IR_1 + IR_2 + IR_3$$
$$\text{Or, } R_{eq} = R_1 + R_2 + R_3$$



(a) Series Connection



(b) Equivalent

Figure 1.5: Series connection of resistor

Properties of series circuits

1. Same current passes through all the resistors.
2. Summation of voltage drops across the series connected resistors equals to the supply voltage, V i.e.$V = V_1 + V_2 + \ldots + V_n = \sum^n V_i$.
3. Equivalent resistance of the whole combination is equal to the sum of individual resistances i.e.$R_{eq} = R_1 + R_2 + \ldots + R_n = \sum^n R_i$.
4. Maximum voltage drop takes place across the resistor having maximum resistance.
5. If $R_1 = R_2 = \ldots = R_n = R$ then $R_{eq} = nR$.

Voltage divider relationships

$$\begin{aligned} V_1 &= IR_1 = \frac{R_1}{R_{eq}}V = \frac{R_1}{R_1 + R_2 + R_3}V \\ V_2 &= IR_2 = \frac{R_2}{R_{eq}}V = \frac{R_2}{R_1 + R_2 + R_3}V \\ V_3 &= IR_3 = \frac{R_3}{R_{eq}}V = \frac{R_3}{R_1 + R_2 + R_3}V \end{aligned}$$

These relationships are helpful to find the potential drop across a particular component when the supply voltage is given.
If $R_1 = R_2 = R_3 = R$, then $V_1 = V_2 = V_3 = \frac{V}{3}$.
In general, if $R_1 = R_2 = \ldots = R_n = R$, then $V_1 = V_2 = \ldots = V_n = \frac{V}{n}$.

Example 1.11 *Three resistors 2Ω, 3Ω and 5Ω are connected in series across a 20V supply system. What will be the voltage across each resistor?*

Solution Current flowing through the circuit

$$I = \frac{20}{2 + 3 + 5} = 2A$$

Voltage across 2Ω resistor $V_1 = IR_1 = 2 \times 2 = 4\ volts$

Voltage across 3Ω resistor $V_2 = IR_2 = 2 \times 3 = 6\ volts$

Voltage across 5Ω resistor $V_3 = IR_3 = 2 \times 5 = 10\ volts$

Example 1.12 *Three resistors 3Ω, 6Ω and 9Ω are connected in series across a 90V supply system. What will be the power absorbed by 3Ω resistor?*

Solution Current flowing through the circuit

$$I = \frac{90}{3 + 6 + 9} = 5A$$

Power absorbed by 3Ω resistor

$$P = I^2R = 5^2 \times 3 = 75W$$

Example 1.13 *The element of a 500W electric press is designed for use on 200V, 50Hz supply. What value of resistance is needed to be connected in series in order that the press can be operated at 230V ac supply.*
[Punjab Technical University,Basic Electrical and Electronics Engineering, December, 2004]

Solution Rated current of the press $I = \frac{P}{V} = \frac{500}{200} = 2.5A$

Extra voltage applied $V_R = 230 - 200 = 30V$ must be dropped across the external resistance.

External resistance to be connected in series $R_{ext} = \frac{V_R}{I} = \frac{30}{2.5} = 12\Omega$

Example 1.14 *A 100V, 60W bulb is to be operated from 220V supply. What resistance must be connected in series with the bulb to glow normally?*

Solution Rated current of the bulb $I = \frac{P}{V} = \frac{60}{100} = 0.6A$

Extra voltage applied $V_R = 220 - 100 = 120V$ must be dropped across the external resistance.

External resistance to be connected in series $R_{ext} = \frac{V_R}{I} = \frac{120}{0.6} = 200\Omega$

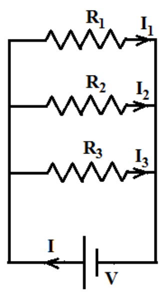
1.4.2 Parallel Connection of Resistors

Resistors are said to be connected in parallel when they are connected between the same two points i.e. the same voltage drop appears across them. Current drawn by the combination must be the sum of the currents drawn by the each resistor. Figure 1.6a shows three resistors R_1, R_2 and R_3 connected in parallel. Current drawn by them are I_1, I_2 and I_3 respectively. Applying Kirchhoff's current law:

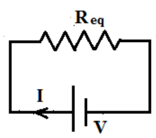
$$I = I_1 + I_2 + I_3$$
$$\text{Or, } \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$
$$\text{Or, } \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Or, we can write,

$$I = I_1 + I_2 + I_3$$
$$\text{Or, } VG_{eq} = VG_1 + VG_2 + VG_3$$
$$\text{Or, } G_{eq} = G_1 + G_2 + G_3 = \frac{1}{R_{eq}}$$



(a) Parallel connection



(b) Equivalent

Figure 1.6: Parallel connection of resistor

Properties of parallel circuits

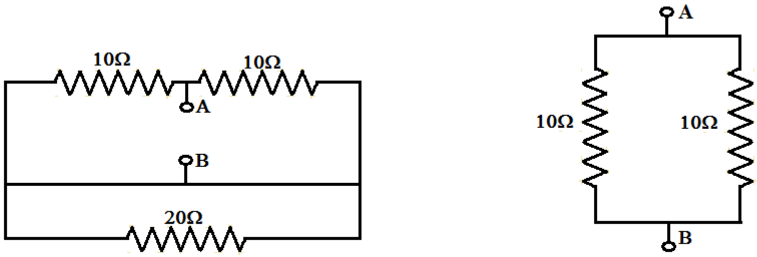
1. Voltage across all the resistors is same.
2. Summation of current passing through the parallel connected resistors equals to the supply current, I i.e.$I = I_1 + I_2 + \ldots + I_n = \sum^n I_i$.
3. Equivalent conductance of the whole combination is equal to the sum of individual conductances i.e.$G_{eq} = G_1 + G_2 + \ldots + G_n = \sum^n G_i$.
4. Maximum current passing through the resistor having maximum conductance or least resistance.
5. If $G_1 = G_2 = \ldots = G_n = G$ then $G_{eq} = nG$ i.e. $R_{eq} = 1/G_{eq} = R/n$

Current divider relationships

$$I_1 = VG_1 = \frac{G_1}{G_{eq}} I = \frac{G_1}{G_1 + G_2 + G_3} I$$
$$I_2 = VG_2 = \frac{G_2}{G_{eq}} I = \frac{G_2}{G_1 + G_2 + G_3} I$$
$$I_3 = VG_3 = \frac{G_3}{G_{eq}} I = \frac{G_3}{G_1 + G_2 + G_3} I$$

These relationships are helpful to find the current through a particular branch when the supply current is known.
If $R_1 = R_2 = R_3 = R$, then $G_1 = G_2 = G_3 = G$. We get $I_1 = I_2 = I_3 = \frac{I}{3}$
In general, if $R_1 = R_2 = \ldots = R_n = R$, then $G_1 = G_2 = \ldots = G_n = G$. We get $I_1 = I_2 = \ldots = I_n = \frac{I}{n}$

Example 1.15 *Find equivalent resistance between A and B in the network shown in Figure 1.8a.*



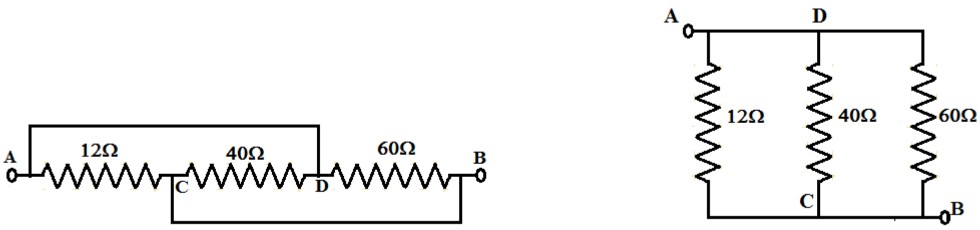
(a) (b)

Figure 1.7: Circuit diagrams of Example 1.15

Solution From Figure **??**

$$R_{AB} = 10||10 = \frac{10 \times 10}{10 + 10} = 5\Omega$$

Example 1.16 *Find equivalent resistance between A and B in the network shown in Figure* **??**.



(a) (b)

Figure 1.8: Circuit diagrams of Example 1.16

Solution

$$R_{AB} = 12||40||60 = 12||\frac{40 \times 60}{40 + 60} = 12||24 = \frac{12 \times 24}{12 + 24} = 8\Omega$$

Example 1.17 *Two resistors are connected in parallel across a 200V dc supply. The total current taken is 20 A and power dissipated in one of the resistors is 2000W. What is the resistance of each element?*

Solution

$$\begin{aligned} R_1 &= \frac{V^2}{P} = \frac{200^2}{2000} = 20\Omega \\ R_{eq} &= \frac{V}{I} = \frac{200}{20} = 10\Omega \\ 10 &= \frac{R_1 R_2}{R_1 + R_2} = \frac{20R_2}{20 + R_2} \\ R_2 &= 20\Omega \end{aligned}$$

Example 1.18 *Two coils are connected in parallel across a 120V dc supply, take 10 A from the supply. Power dissipated in one coil is 400W. What is the resistance of each coil?*

Solution

$$\begin{aligned} R_1 &= \frac{V^2}{P} = \frac{120^2}{400} = 36\Omega \\ R_{eq} &= \frac{V}{I} = \frac{120}{10} = 12\Omega \\ 12 &= \frac{R_1 R_2}{R_1 + R_2} = \frac{36R_2}{36 + R_2} \\ R_2 &= 18\Omega \end{aligned}$$

Example 1.19 *For the circuit shown in Figure 1.9 find the total resistance and battery current.*

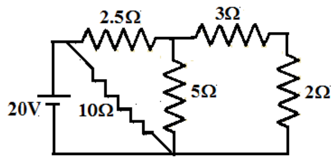
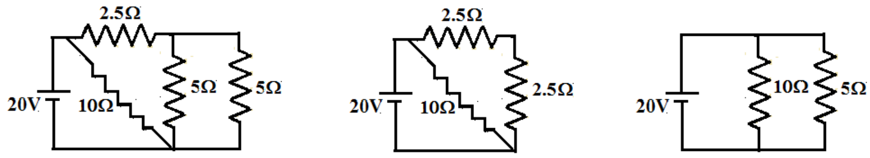


Figure 1.9: Circuit diagram for Example 1.19



(a) (b) (c)

Figure 1.10: Circuit diagram for Example 1.19

Solution Resistances 3Ω and 2Ω are connected in series. By adding them we get circuit shown in Figure 1.10a. Now, two 5Ω resistances are in parallel. Their equivalent resistance will be 2.5Ω shown in Figure 1.10b. Now, two 2.5Ω resistances are in series. The circuit reduces to Figure 1.10c.

$$R_{eq} = 10||5 = \frac{10 \times 5}{10+5} = \frac{10}{3} = 3.33\Omega$$

Battery current $= \frac{20\times3}{10} = 6A$

Example 1.20 *For the circuit shown in Figure 1.11 find the value of R for which the total current taken is 9A. Determine the branch currents I_1 and I_2.*

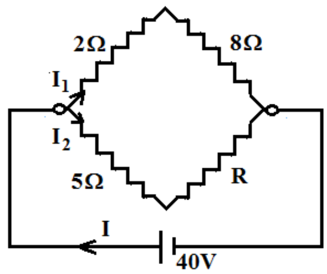


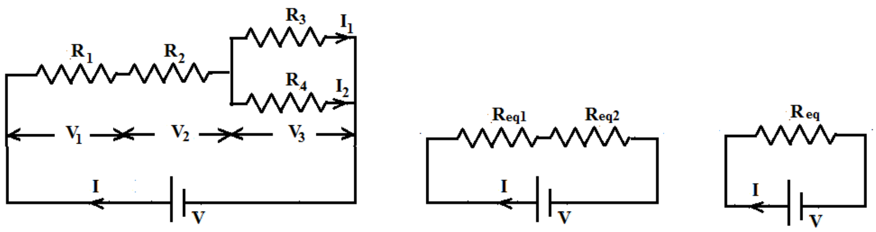
Figure 1.11: Circuit diagram for Example 1.20

Solution

$$\begin{aligned}
R_{eq} &= \frac{40}{9}\Omega \\
R_{eq} &= (2+8)||(5+R) = \frac{10\times(5+R)}{15+R}\Omega \\
\text{Or, } R &= 3\Omega \\
I_1 &= \frac{5+3}{5+3+2+8}\times 9 = \frac{8}{18}\times 9 = 4A \\
I_2 &= 9-4 = 5A
\end{aligned}$$

1.4.3 Series-Parallel connection

In some complex circuits both series and parallel connections are present as shown in Figure 1.12.



(a) Series-Parallel connection (b) Simplied series connection (c) Equivalent

Figure 1.12: Series-Parallel connected Resistors and their equivalent resistor

Steps:

1. Determine equivalent resistance of the series connected part ($R_{eq1} = R_1 + R_2$).
2. Determine equivalent resistance of the parallel connected part ($\frac{1}{R_{eq2}} = \frac{1}{R_1} + \frac{1}{R_2}$).
3. Now add up them to get total equivalent resistance ($R_{eq} = R_{eq1} + R_{eq2}$).

Example 1.21 *A resistance of* 10Ω *is connected in series with two resistances each of* 15Ω *in parallel. What resistance must be shunted across this branch so that the total current taken shall be 1.5A when connected across a 20V dc supply?*

Solution

$$
\begin{aligned}
R_{eq1} &= 10 + 15||15 = 10 + \frac{15}{2} = \frac{35}{2}\Omega \\
R_{eq} &= \frac{20}{1.5} = \frac{40}{3}\Omega \\
R_{eq} &= \frac{R_{eq1} \times R_{Sh}}{R_{eq1} + R_{Sh}} \\
\text{Or, } \frac{40}{3} &= \frac{\frac{35}{2} \times R_{Sh}}{\frac{35}{2} + R_{Sh}} \\
\text{Or, } R_{Sh} &= 56\Omega
\end{aligned}
$$

Example 1.22 *In the circuit shown in Figure 1.13a, calculate the power dissipated in each resistor and the reading of a voltmeter connected across* 7Ω *resistor.*

(a) (b)

Figure 1.13: Circuit diagrams for Example 1.22

Solution

$$
\begin{aligned}
R_{eq} &= 7 + \frac{12 \times 24}{12 + 24} = 15\Omega \\
I &= \frac{180}{15} = 12A \\
I_1 &= \frac{24}{12 + 24} \times 12 = 8A \\
I_2 &= 12 - 8 = 4A
\end{aligned}
$$

$$\begin{aligned}
\text{Power dissipated in } 7\Omega \text{ resistor, } P_{7\Omega} &= 12^2 \times 7 = 1008W \\
\text{Power dissipated in } 12\Omega \text{ resistor, } P_{12\Omega} &= 8^2 \times 12 = 768W \\
\text{Power dissipated in } 24\Omega \text{ resistor, } P_{24\Omega} &= 4^2 \times 24 = 384W \\
\text{Voltmeter reading connected across } 7\ \Omega \text{ resistor, } V_{7\Omega} &= 12 \times 7 = 84V
\end{aligned}$$

Example 1.23 *If 9 V is applied across A and B (Figure 1.14), calculate the total current and the value of a series resistance to halve the total current.*

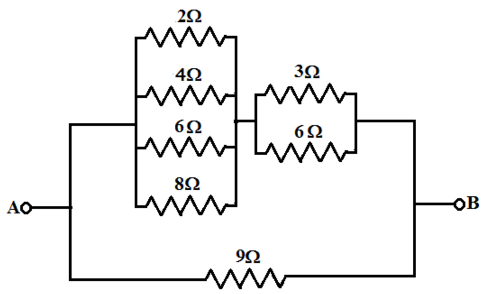


Figure 1.14: Circuit diagram for Example 1.23

Solution

$$\begin{aligned}
Y_1 &= \frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8} = 1.0417\mho \\
R_1 &= 0.96\Omega \\
R_2 &= \frac{3\times 6}{3+6} = 2\Omega \\
R_{eq} &= \frac{3\times 2.96}{3+2.96} = 1.4899\Omega \\
I &= \frac{9}{1.4899} = 6.0407A
\end{aligned}$$

Value of a series resistance to halve the total current

$$R_{ext} = R_{eq} = 1.4899\Omega$$

Example 1.24 *Twelve identical wires, each of resistance R are connected to form a cube as shown in Figure 1.15. If a current of 10A enters into the cube at the corner A and leaves it diagonally from the opposite corner G. What will be the potential difference between these two corners for $R = 6\Omega$?*

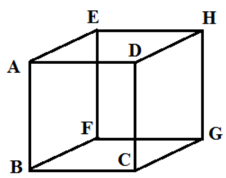


Figure 1.15: Circuit diagram for Example 1.24

Solution As all the resistances are identical, current entering the corner A will be equally divided into three sides AB, AD, AE. Hence, B,D and E are at equal potential. Similarly currents entering the node G are equal and C,F,H are at equal potential. At point B current in branch AB divides equally to flow into BC and BF, at D the branch current of AD divides equally to flow into DH and DC and at E the branch current of AE divides equally to flow into EH and EF. The equivalent circuit of this cube is shown in Figure 1.16.

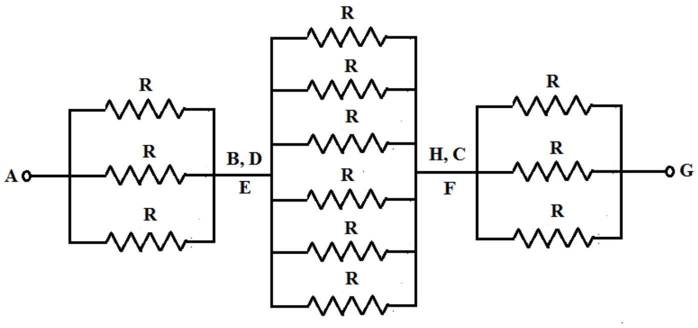


Figure 1.16: Circuit diagram for Example 1.24

$$
\begin{aligned}
R_{AD} &= R_{CG} = \frac{R}{3} \\
R_{DC} &= \frac{R}{6} \\
R_{eq} &= R_{AD} + R_{DC} + R_{CG} = \frac{R}{3} + \frac{R}{6} + \frac{R}{3} = \frac{5R}{6} = 5\Omega \\
V_{AG} &= 10 \times 5 = 50 Volts
\end{aligned}
$$

Example 1.25 *Find equivalent resistance between A and B in the network shown in Figure 1.17a.*

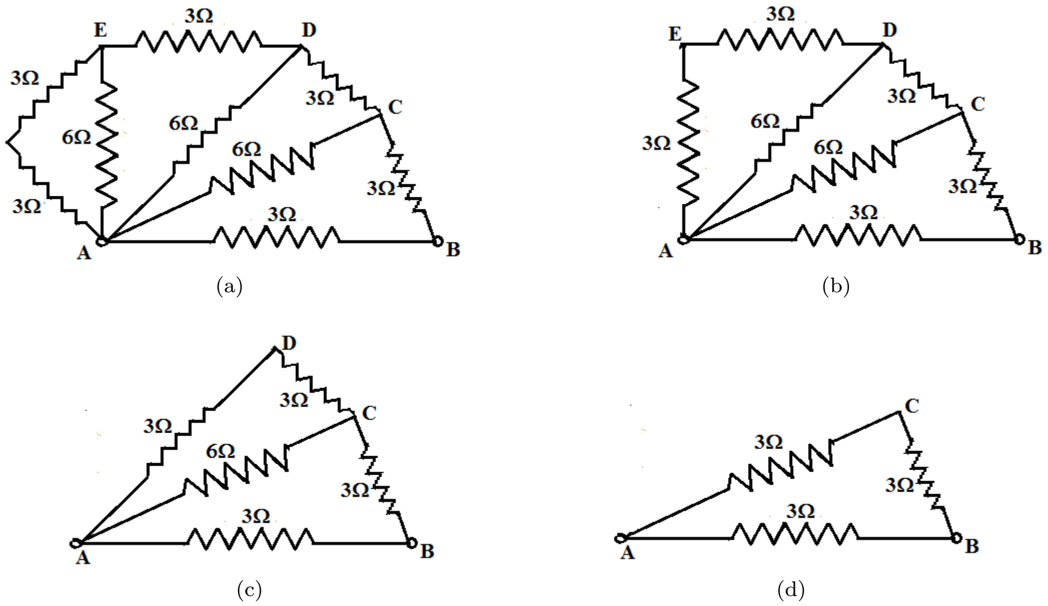


Figure 1.17: Circuit diagrams of Example 1.25

Solution We know,

$$
\begin{aligned}
(3+3)||6 &= \frac{6 \times 6}{6+6} = 3\Omega \\
R_{AB} = 3||(3+3) &= \frac{3 \times 6}{3+6} = 2\Omega
\end{aligned}
$$

Example 1.26 *Find equivalent resistance between A and B in the network shown in Figure 1.18a.*

Solution From Figure 1.18b

$$
R_{AB} = (10+10)||(10+10) = 20||20 = \frac{20 \times 20}{10+20} = 10\Omega
$$

Example 1.27 *Find equivalent resistance between A and B in the network shown in Figure 1.19a.*

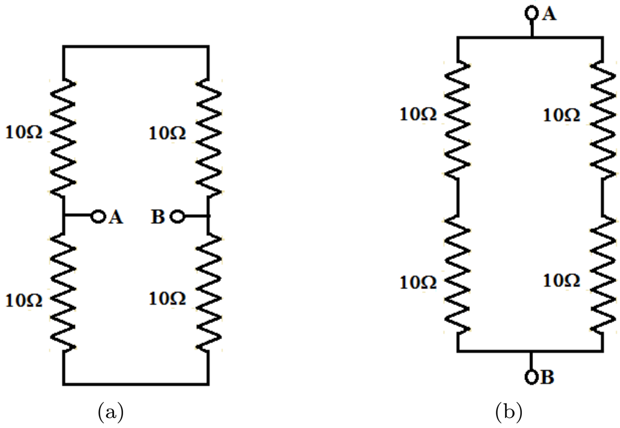


Figure 1.18: Circuit diagrams of Example 1.26

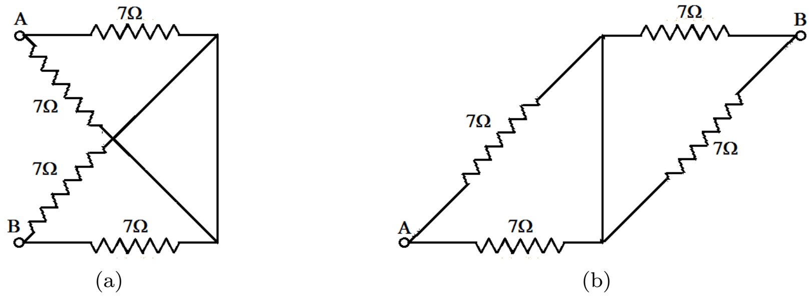


Figure 1.19: Circuit diagrams of Example 1.27

Solution From Figure 1.19b

$$R_{AB} = (7||7) + (7||7) = 3.5 + 3.5 = 7\Omega$$

Example 1.28 *Find equivalent resistance between A and B in the network shown in Figure 1.20a.*

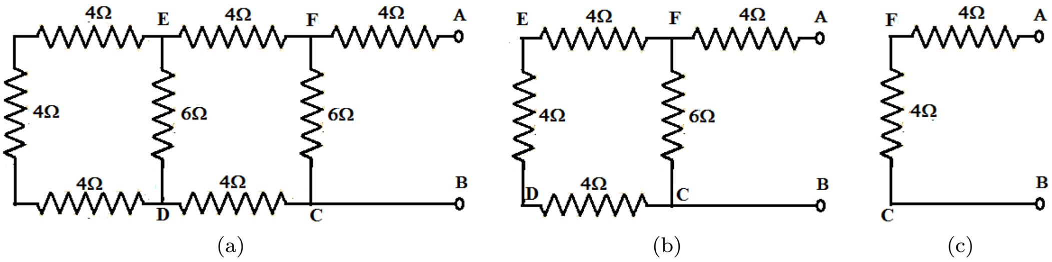


Figure 1.20: Circuit diagrams of Example 1.28

Solution

$$(4 + 4 + 4)||6 = \frac{12 \times 6}{12 + 6} = \frac{72}{18} = 4\Omega$$

From Figure 1.20c

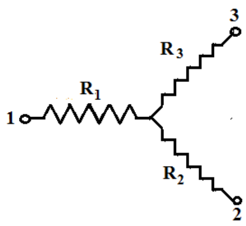
$$R_{AB} = 4 + 4 = 8\Omega$$

1.4.4 Star and Delta Connection of Resistors

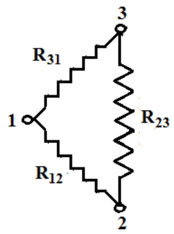
Apart from series and parallel connection three resistors may be connected in star or delta fashion as shown in Figures 1.21a and 1.21b respectively.

Star-Delta transformation technique

Sometimes star to delta or delta to star transformation makes the circuit simpler to deal with.



(a)



(b) Delta connection

Figure 1.21: Star connected Resistors and their equivalent delta connected resistors

If the resistors connected in star shown in Figure 1.21a is equivalent to the delta connected resistors shown in Figure 1.21b, they should have same equivalent resistances between the same pair of terminals. Hence, we can write

$$R_1 + R_2 = R_{12}||(R_{23} + R_{31}) = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \tag{1.1}$$

$$R_2 + R_3 = R_{23}||(R_{31} + R_{12}) = \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \tag{1.2}$$

$$R_3 + R_1 = R_{31}||(R_{12} + R_{23}) = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \tag{1.3}$$

Adding equations 1.1, 1.2, 1.3

$$R_1 + R_2 + R_3 = \frac{R_{12}R_{23} + R_{23}R_{31} + R_{31}R_{12}}{R_{12} + R_{23} + R_{31}} \tag{1.4}$$

Delta to star relationships: Subtracting equations 1.2, 1.3, 1.1 from equation no. 1.4, we get

$$R_1 = \frac{R_{31}R_{12}}{R_{12} + R_{23} + R_{31}} \tag{1.5}$$

$$R_2 = \frac{R_{12}R_{23}}{R_{12} + R_{23} + R_{31}} \tag{1.6}$$

$$R_3 = \frac{R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} \tag{1.7}$$

Each resistor of star connection is the product of adjacent delta resistors divided by the sum of all the three delta resistors. To find R_1 product R_{12} and R_{31} should be divided bythe sum of all the three delta resistors.
If $R_{12} = R_{23} = R_{31} = R$ then $R_1 = R_2 = R_3 = R/3$.

Star to delta relationships: Multiplying equation no. 1.5 with equation no. 1.6, equation no. 1.6 with equation no. 1.7,equation no. 1.7 with equation no. 1.5 and adding the products we get

$$\begin{aligned} R_1R_2 + R_2R_3 + R_3R_1 &= \frac{R_{12}^2R_{23}R_{31} + R_{12}R_{23}^2R_{31} + R_{12}R_{23}R_{31}^2}{(R_{12} + R_{23} + R_{31})^2} \\ &= \frac{R_{12}R_{23}R_{31}}{R_{12} + R_{23} + R_{31}} \end{aligned} \tag{1.8}$$

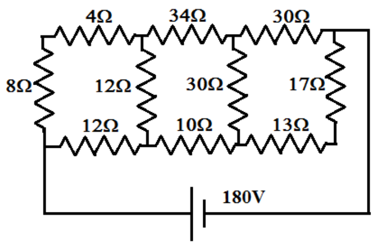
Dividing equation no. 1.8 with equations 1.7, 1.5, 1.6, we get

$$R_{12} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} = R_1 + R_2 + \frac{R_1R_2}{R_3} \tag{1.9}$$

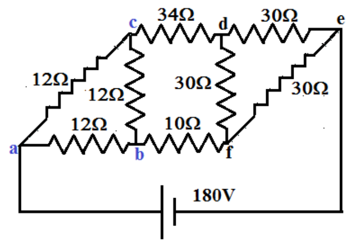
$$R_{23} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} = R_2 + R_3 + \frac{R_2R_3}{R_1} \tag{1.10}$$

$$R_{31} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} = R_3 + R_1 + \frac{R_3R_1}{R_2} \tag{1.11}$$

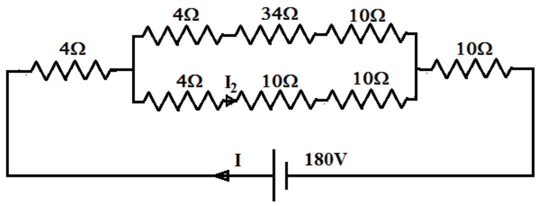
Each resistor of delta connection is the sum of all possible products of three star resistances two taken at a time divided by the opposite resistor. While finding R_{12} the sum of products should be divided by R_3.
If $R_1 = R_2 = R_3 = R$ then $R_{12} = R_{23} = R_{31} = 3R$.



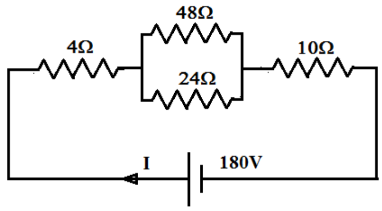
(a) Original circuit



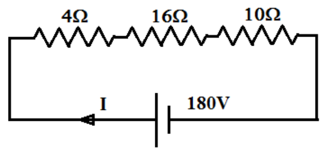
(b)



(c)



(d)



(e) Simplified circuit

Figure 1.22: Circuit diagrams of Example 1.29

Example 1.29 *In the circuit shown in Figure 1.22a determine the current flowing through* 10Ω *resistor.*

Solution a-b-c and d-e-f are two deltas present in the circuit (Figure 1.22b. Convert the deltas to star to simplify the circuit (Figure 1.22c). A series parallel circuit is obtained (Figure 1.22d), simplify this to get a simple series circuit(Figure 1.22e).

$$
\begin{aligned}
R_{eq} &= 4 + 48||24 + 10 = 14 + \frac{48 \times 24}{48 + 24} = 14 + 16 = 30\Omega \\
I &= \frac{V}{R_{eq}} = \frac{180}{30} = 6A \\
I_2 &= \frac{48}{48 + 24} \times 6 = 4A
\end{aligned}
$$

Example 1.30 *In the circuit shown in Figure 1.23a, determine*

1. *the current drawn from the supply*
2. *currents flowing through BC, CA and AB.*
3. *currents flowing through Cc, Aa and Bb.*
4. *currents flowing through bc, ca and ab.*

Solution a-b-c and A-B-C are two deltas present in the circuit (Figure 1.23a. Convert the delta a-b-c to star (Figure 1.23b). Add the resistance 2Ω and 1/3Ω (Figure 1.23c) and convert the new star to delta to simplify the circuit. A series parallel circuit is obtained (Figure 1.23d), simplify this to get a simple series circuit(Figure 1.23e).

$$
\begin{aligned}
R_{eq} &= \frac{7}{8} + \frac{7}{8}||\frac{7}{8} = \frac{7}{12}\Omega \\
I &= \frac{V}{R_{eq}} = \frac{7 \times 12}{7} = 12A \\
I_1 &= \frac{1}{3} \times 12 = 4A \\
I_2 &= \frac{2}{3} \times 12 = 8A
\end{aligned}
$$

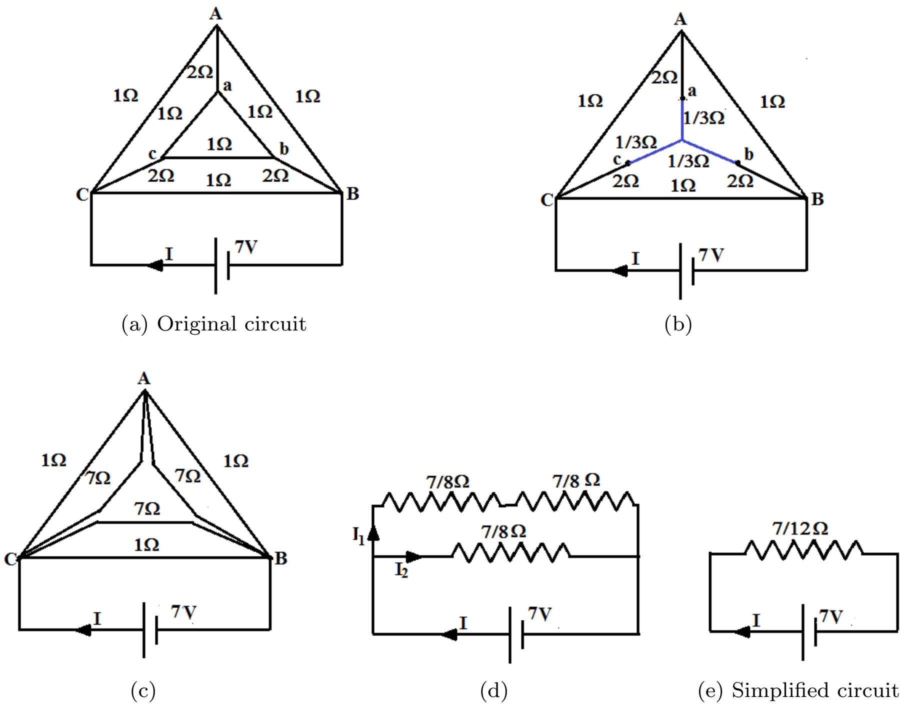


Figure 1.23: Circuit diagrams of Example 1.30

Current through BC= $\frac{7}{8} \times 8 = 7A$
Current through AB= Current through CA $= \frac{7}{8} \times 4 = 3.5A$
Current through Cc =Current through Bb $== 12 - 7 - 3.5 = 1.5A$
Current through Aa=0A
Current through bc= $\frac{2}{3} \times 1.5 = 1A$
Current through ca=Current through ab= $\frac{1}{3} \times 1.5 = 0.5A$

Example 1.31 *Find equivalent resistance between A and B in the network shown in Figure 1.24a.*

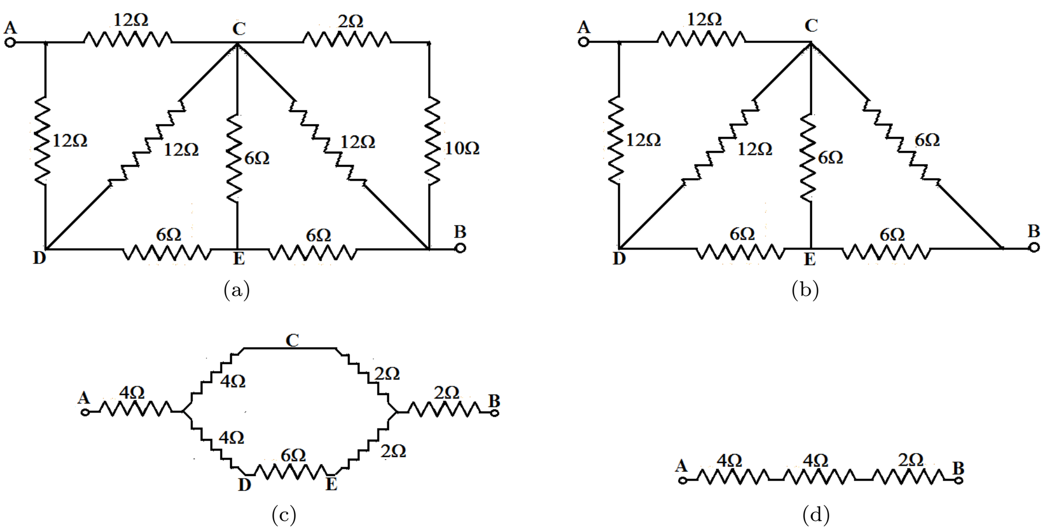


Figure 1.24: Circuit diagrams of Example 1.31

Solution We know that a delta comprising of three equal resistances $R\Omega$ may be converted to a star of $\frac{R}{3}\Omega$. Let us convert deltas ACE and BCD into stars.

From Figure 1.24c, we can write

$$(4+2)||(4+6+2) = \frac{6\times 12}{6+12} = 4\Omega$$

From Figure 1.24d,

$$R_{AB} = 4+4+2 = 10\Omega$$

Example 1.32 *Find equivalent resistance between A and B in the network shown in Figure 1.25a.*

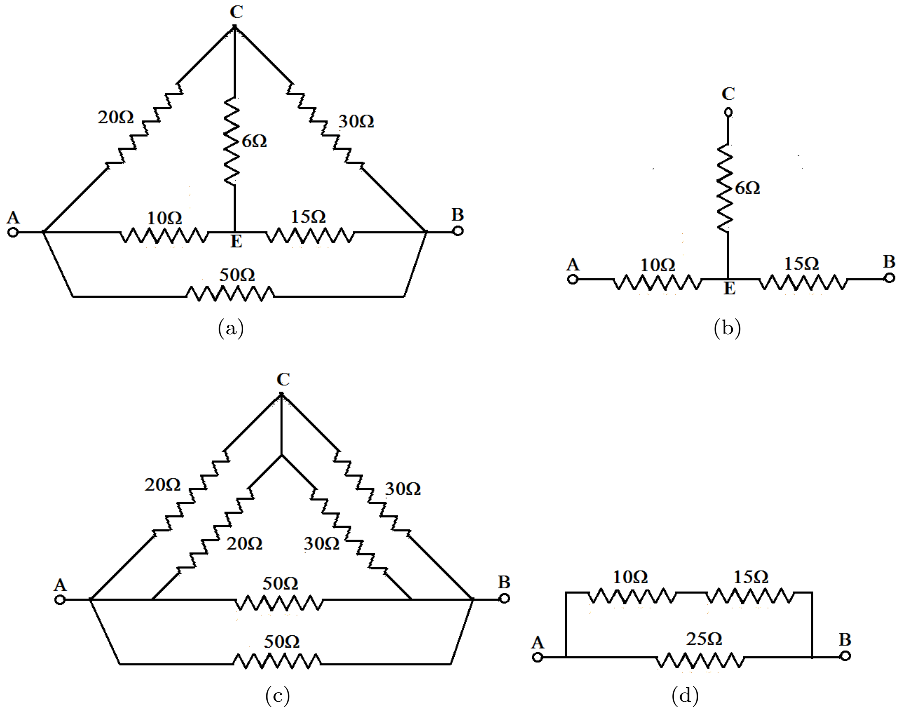


Figure 1.25: Circuit diagrams of Example 1.32

Solution Let first convert the star ABC to delta. From Figure 1.25b

$$R_A = 10\Omega$$
$$R_B = 15\Omega$$
$$R_C = 6\Omega$$
$$R_{AB} = 10+15+\frac{10\times 15}{6} = 50\Omega$$
$$R_{BC} = 15+6+\frac{15\times 6}{10} = 30\Omega$$
$$R_{CA} = 10+6+\frac{10\times 6}{15} = 20\Omega$$

From Figure 1.25d

$$R_{eq} = (10+15)||25 = 25||25 = \frac{25\times 25}{25+25} = 12.5\Omega$$

Example 1.33 *Find equivalent resistance between A and B in the network shown in Figure 1.26a.*

Solution Let's convert the inside star ABC having resistances R. The delta obtained will have each branch resistances $3R$ as shown in Figure 1.26b.

$$3R||3R = \frac{3R\times 3R}{3R+3R} = 1.5R$$

From Figure 1.26c

$$R_{eq} = (1.5R+1.5R)||1.5R = 3R||1.5R = \frac{3R\times 1.5R}{3R+1.5R} = R\Omega$$

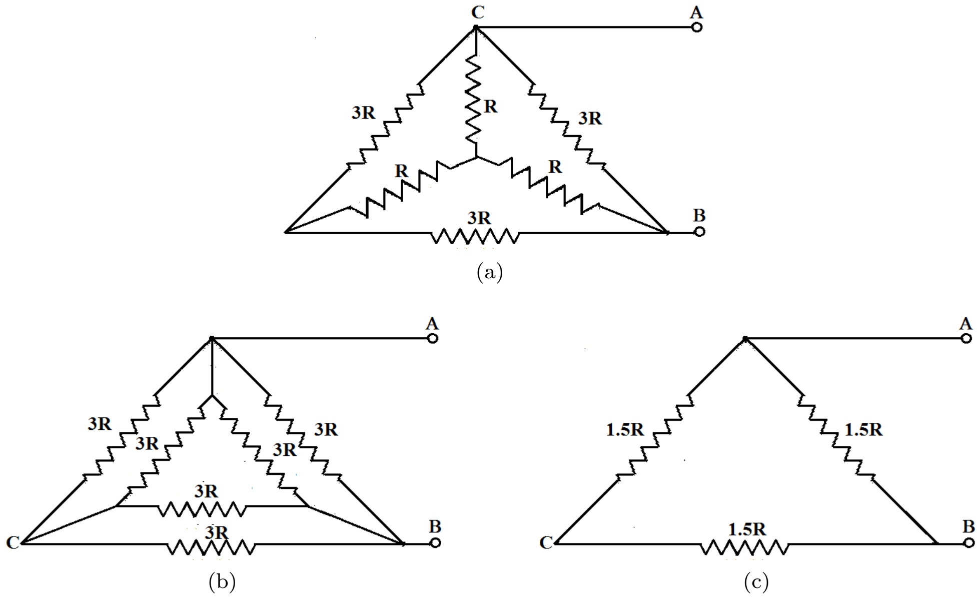


Figure 1.26: Circuit diagrams of Example 1.33

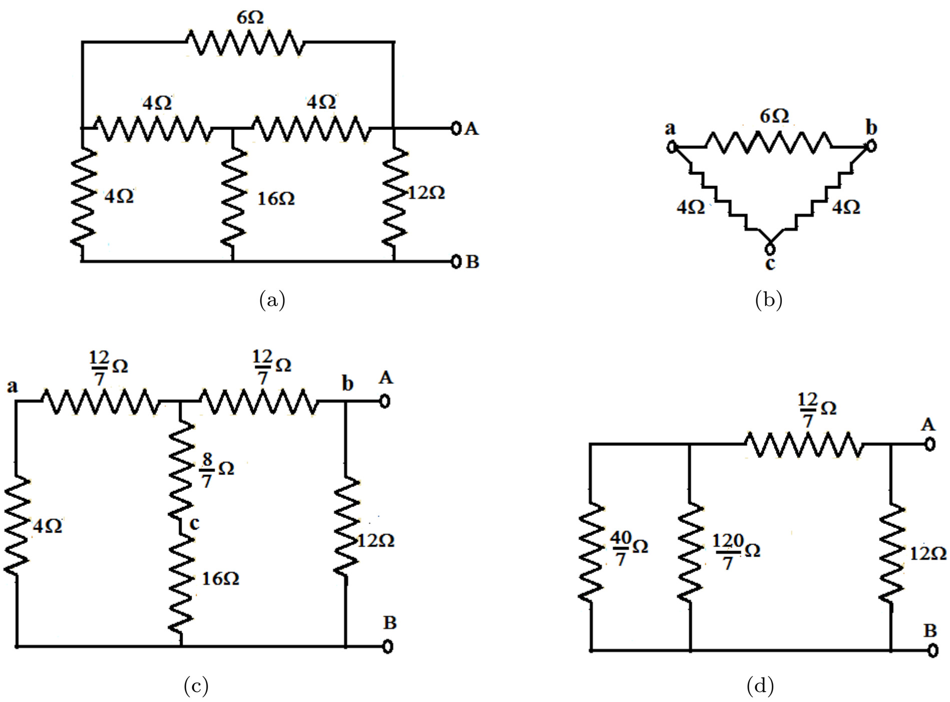


Figure 1.27: Circuit diagrams of Example 1.34

Example 1.34 *Find equivalent resistance between A and B in the network shown in Figure 1.27a.*

Solution Converting the delta abc to star when

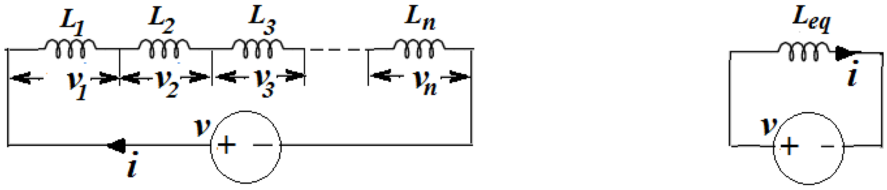
$$R_{ab} = 6\Omega$$
$$R_{bc} = Rca = 4\Omega$$
$$R_a = \frac{6 \times 4}{6+4+4} = \frac{24}{14} = \frac{12}{7}\Omega$$
$$R_b = \frac{4 \times 6}{6+4+4} = \frac{24}{14} = \frac{12}{7}\Omega$$
$$R_c = \frac{4 \times 4}{6+4+4} = \frac{16}{14} = \frac{8}{7}\Omega$$
$$R_{eq} = 12|| \left[\frac{12}{7} + \left(16 + \frac{8}{7}\right) || \left(4 + \frac{12}{7}\right)\right] = 12|| \left[\frac{12}{7} + \frac{120}{7}||\frac{40}{7}\right]$$
$$= 12|| \left[\frac{12}{7} + \frac{\frac{120}{7} \times \frac{40}{7}}{\frac{120}{7} + \frac{40}{7}}\right] = 12|| \left[\frac{12}{7} + \frac{30}{7}\right] = 12||6 = 4\Omega$$

1.4.5 Series Connection of Inductors

When inductors are series connected and they are not mutually coupled, same current flows through all of them but potential difference across them could be different depending upon the value of inductances.
$v_1 = L_1\frac{di}{dt}$, $v_2 = L_2\frac{di}{dt}$, $v_3 = L_3\frac{di}{dt}$
Supply voltage,

$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

where L_{eq} is the equivalent inductor.
If $L_1 = L_2 = L_3 = L$, $L_{eq} = 3L$



(a) (b) Equivalent

Figure 1.28: Series connection of inductors

Example 1.35 *Four inductors of 2 mH, a 3 mH, 5 mH and 10mH are connected in series. What will be the total inductance ?*

Solution The total inductance is

$$L_{eq} = L_1 + L_2 + L_3 + L_4 = 2 + 3 + 5 + 10 = 20mH$$

1.4.6 Parallel Connection of Inductors

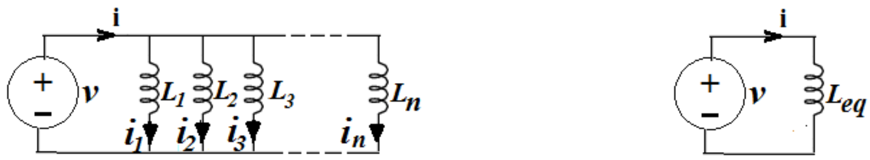
When inductors are connected in parallel, same potential difference appears across them but current flowing would be different depending upon the values of inductors. If no mutual coupling exists

$$v = L_1\frac{di_1}{dt}, \quad v = L_2\frac{di_2}{dt}, \quad v = L_3\frac{di_3}{dt}$$

Total current, $i = i_1 + i_2 + i_3 = (\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3})\int vdt$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

where L_{eq} is the equivalent inductor.
If $L_1 = L_2 = L_3 = L$, $L_{eq} = L/3$
If inductors are connected in parallel, overall inductor reduces whereas if connected in series overall inductor increases.



(a) (b) Equivalent

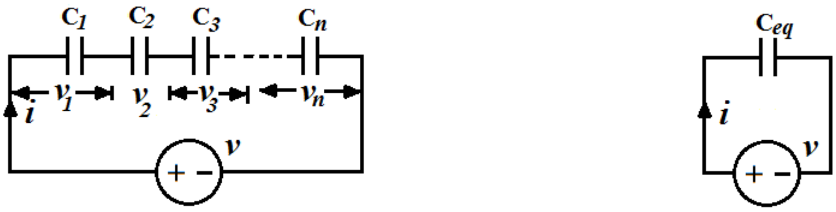
Figure 1.29: Parallel connection of inductors

1.4.7 Series Connection of Capacitors

When capacitors are series connected same current flows through all of them but potential difference across them could be different depending upon the value of capacitances.
$Q = C_1V_1$, $Q = C_2V_2$, $Q = C_3V_3$
Supply voltage, $V = V_1 + V_2 + V_3 = (\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3})Q$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{C_{eq}}$$

where C_{eq} is the equivalent capacitor.
If $C_1 = C_2 = C_3 = C$, $C_{eq} = C/3$



(a) (b) Equivalent

Figure 1.30: Series connection of capacitors

Example 1.36 *One $2\mu F$ capacitor is connected in series with another capacitor of unknown value. This combination is charged from a 24 V source. The $2\mu F$ capacitor is charged to 16 V. What is the value of the unknown capacitor ?*

Solution Supply voltage,

$$\begin{aligned} Q &= C_1V_1 = 2 \times 10^{-6} \times 16 = 32\mu Coulomb \\ V_2 &= V_s - V_1 = 32 - 16 = 8V \\ C_2 &= \frac{Q}{V_2} = \frac{32 \times 10^{-6}}{8} = 4\mu F \end{aligned}$$

The value of the unknown capacitor is $4\mu F$

Example 1.37 *Three capacitors of values $4\mu F$, $6\mu F$ and $12\mu F$are connected in series to a $150V$ dc supply. Calculate: (a) the equivalent capacitance; (b) the charge stored on each capacitor; (c) the voltage across each capacitor; (d) the energy stored in each capacitor.*

Solution

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{4} + \frac{1}{6} + \frac{1}{12} = \frac{1}{2}/\mu F$$

Equivalent capacitance, $C_{eq} = 2\mu F$

Charge stored on each capacitor:

$Q = Q_1 = Q_2 = Q_3 = C_{eq}V = 2 \times 10^{-6} \times 150 = 0.3mC$

Voltage across each capacitor:

$$V_1 = \frac{Q}{C_1} = \frac{0.3 \times 10^{-3}}{4 \times 10^{-6}} = 75V$$

$$V_2 = \frac{Q}{C_2} = \frac{0.3 \times 10^{-3}}{6 \times 10^{-6}} = 50V$$

$$V_3 = \frac{Q}{C_3} = \frac{0.3 \times 10^{-3}}{12 \times 10^{-6}} = 25V$$

Energy stored in each capacitor:

$$E_1 = \frac{1}{2}CV_1^2 = \frac{1}{2} \times 4 \times 10^{-6} \times 75^2 = 0.01125J$$

$$E_2 = \frac{1}{2}CV_2^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 50^2 = 0.0075J$$

$$E_3 = \frac{1}{2}CV_3^2 = \frac{1}{2} \times 12 \times 10^{-6} \times 25^2 = 0.00375J$$

1.4.8 Parallel connection of Capacitors

When capacitors are connected in parallel same potential difference appears across them but charge stored could be different depending upon the value of capacitances.

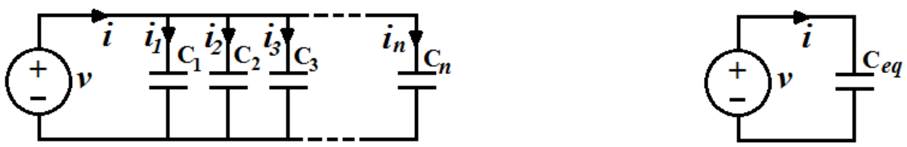
$Q_1 = C_1V$, $Q_2 = C_1V$, $Q_3 = C_3V$

Total charge stored, $Q = Q_1 + Q_2 + Q_3 = (C_1 + C_2 + C_3)V$

Equivalent capacitor, $C_{eq} = \frac{Q}{V} = C_1 + C_2 + C_3$

If $C_1 = C_2 = C_3 = C$, $C_{eq} = 3C$

If capacitances are connected in parallel, overall capacitance increases where as if connected in series overall capacitance reduces.



(a) (b) Equivalent

Figure 1.31: Parallel connection of capacitors

Example 1.38 *Four capacitors of* $10\mu F$*,* $20\mu F$*,* $40\mu F$*, and* $100\mu F$ *are connected in parallel. What will be the total capacitance?*

Solution The total capacitance is

$$C_{eq} = 10 + 20 + 40 + 100 = 170\mu F$$

Example 1.39 *Three capacitors of values* $2\mu F$*,* $8\mu F$ *and* $10\mu F$ *are connected across a 200V dc supply in parallel. Calculate: (a) the resultant capacitance; (b) the total charge; (c) the charge on each capacitor.*

Solution

$$\begin{aligned}
\text{Resultant capacitance, } C_{eq} &= C_1 + C_2 + C_3 = 2 + 8 + 10 = 20\mu F \\
\text{Total charge, } Q &= C_{eq}V = 20 \times 10^{-6} \times 200 = 4mC \\
\text{Charge on each capacitor:} & \\
Q_1 &= C_1V = 2 \times 10^{-6} \times 200 = 0.4mC \\
Q_2 &= C_2V = 8 \times 10^{-6} \times 200 = 1.6mC \\
Q_3 &= C_3V = 10 \times 10^{-6} \times 200 = 2mC
\end{aligned}$$

Example 1.40 *Two capacitors of values $1\mu F$ and $2\mu F$ are connected in parallel. A third capacitor of $12\mu F$ is connected in series with these two in parallel. Calculate the total capacitance. If a 100V battery is connected across the whole circuit, calculate the voltage across each capacitor and the charge held by each capacitor.*

Solution Let C_{eq1} be the equivalent capacitance of the parallel combination of $1\mu F$ and $2\mu F$.

$$\begin{aligned} C_{eq1} &= 1+2 = 3\mu F \\ \text{Total capacitance, } C_{eq} &= \frac{3\times 12}{3+12} = 2.4\mu F \\ Q &= C_{eq}V = 2.4\times 10^{-6}\times 100 = 0.24mC \end{aligned}$$

Voltage across each capacitor:

$$\begin{aligned} V_1 &= V_2 = \frac{Q}{C_{eq1}} = \frac{0.24\times 10^{-3}}{3\times 10^{-6}} = 80V \\ V_3 &= \frac{Q}{C_3} = \frac{0.24\times 10^{-3}}{12\times 10^{-6}} = 20V \end{aligned}$$

Charge held by each capacitor:

$$\begin{aligned} Q_1 &= C_1V_1 = 1\times 10^{-6}\times 80 = 80\mu C \\ Q_2 &= C_2V_2 = 2\times 10^{-6}\times 80 = 160\mu C \\ Q_3 &= C_3V_3 = 12\times 10^{-6}\times 20 = 240\mu C \end{aligned}$$

1.5 Electrical Energy Sources

An electrical energy source is a device which generates electrical energy either in ac or dc form. This may be classified as voltage source and current source. Sources could be dependent or independent type.

1.5.1 Independent Sources

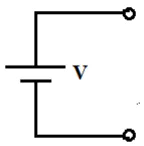
Sources delivering voltage or current which are independent of any other circuit parameters are independent sources.

Independent Voltage Sources

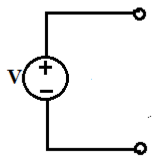
An ideal voltage source (shown in Figures 1.32a and 1.32b) has the following properties:

1. It produces constant output voltage whatever be the value of current drawn from it.
2. It has zero internal resistance.
3. It consumes no power.

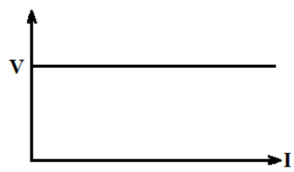
The $V-I$ characteristic of an ideal independent voltage source is shown in Figure 1.32c.



(a) Symbol



(b) Symbol



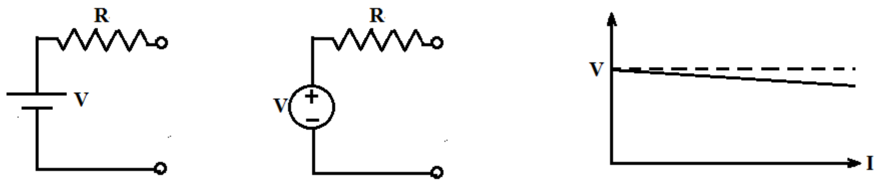
(c) V-I characteristics

Figure 1.32: Ideal voltage source and its V-I characteristics

In reality no ideal voltage sources are available. For a practical voltage source as the output current increases, the voltage drops slightly. This is taken care of by incorporating a small resistance (known as internal resistance) in series with the ideal voltage source (shown in Figures 1.33a and 1.33b).

The V_I characteristic of a practical independent voltage source is shown in Figure 1.33c. DC generators and batteries are the independent dc voltage sources.

Example 1.41 *A practical independent source of 60V can supply a maximum current of 60A. Determine the internal resistance of the source. What would be the load current and voltage when it supplies a load of 30Ω ?*



(a) Symbol (b) Symbol (c) V-I characteristics

Figure 1.33: Practical voltage source and its V-I characteristics

Solution Internal resistance of the source, $r_i = \frac{60}{60} = 1\Omega$
Load current, $I_L = \frac{60}{1+30} = 1.9355A$
Load voltage, $V_L = 1.9355 \times 30 = 58.0645V$

Example 1.42 *A battery of emf 5V and internal resistance* 1Ω *is used to supply power to a variable load resistance* R_L *of range* 100Ω *to* 1000Ω*. Determine percentage change in load voltage* V_L *and load current* I_L*.*

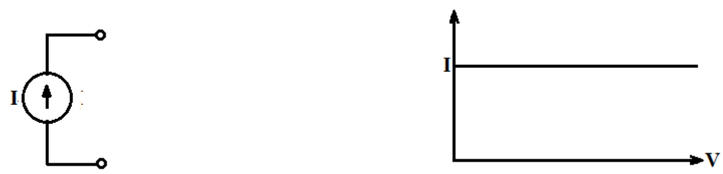
Solution When $R_{L1} = 100\Omega$ $I_{L1} = \frac{5}{1+100} = 0.0495A$
$V_{L1} = E - I_{L1}r_i = 5 - 0.0495 \times 1 = 4.9505V$
When $R_{L2} = 1000\Omega$ $I_{L2} = \frac{5}{1+1000} = 0.005A$
$V_{L2} = E - I_{L2}r_i = 5 - 0.0495 \times 1 = 4.995V$
Change in $I_L = \frac{0.005-0.0495}{0.0495} \times 100\% = -89.9\%$ [-ve sign indicates a decrease.]
Change in $V_L = \frac{4.995-4.9505}{4.9505} \times 100\% = 0.8989\%$ [+ve sign indicates an increase.]

Independent Current Sources

An ideal current source shown in Figure 1.34a has the following properties:

1. It produces constant output current whatever be the value of voltage across it.
2. It has infinite internal resistance.
3. It may deliver infinite power.

The $V - I$ characteristic of an ideal independent current source is shown in Figure 1.34b.



(a) Symbol (b) V-I characteristics

Figure 1.34: Ideal current source and its V-I characteristics

(a) Symbol (b) V-I characteristics

Figure 1.35: Practical current source and its V-I characteristics

In reality no ideal current sources are available. For a practical current source, the output current decreases with increase in the voltage. This is taken care of by incorporating a high resistance (known as internal resistance) in parallel with the ideal current source (shown in Figure 1.35a).

The $V - I$ characteristic of an independent current source is shown in Figure 1.35b. A solar cell is the practical independent dc current source.

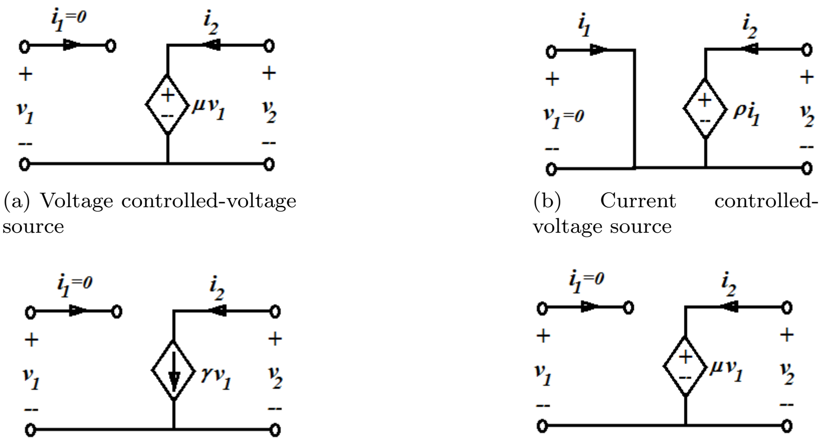
1.5.2 Dependent or controlled sources

Voltage or current delivered by these sources are dependent upon some other voltage or current source present in the circuit. There are four types of dependent sources

1. voltage controlled-voltage source (Figure 1.36a)
2. current controlled-voltage source (Figure 1.36b)
3. voltage controlled-current source (Figure 1.36c)
4. current controlled-current source (Figure 1.36d)

In case of a controlled or dependent voltage source, output is a function of some other circuit variable, it may be a voltage or current, regardless of the current flowing through it.

Similarly a controlled or dependent current source supplies a current which is a function of some other circuit variable, it may be a voltage or current, regardless of its terminal voltage.



(a) Voltage controlled-voltage source

(b) Current controlled-voltage source

(c) Voltage controlled-current source

(d) Current controlled-current source

Figure 1.36: Dependent sources

Dependent sources are found in electronic circuits.

1.6 Source transformation

If the voltage source shown in Figure 1.37a can replace the current source shown in Figure 1.37b and vice versa,

(a) Voltage source

(b) Current source

Figure 1.37: Source transformation

they should deliver same load current (I_L) at the same load voltage (V_L).
From Figure 1.37a we can write

$$V_L = V - I_L R$$

From Figure 1.37b we can write

$$V_L = (I - I_L)R = IR - I_L R$$

Comparing above two equations:

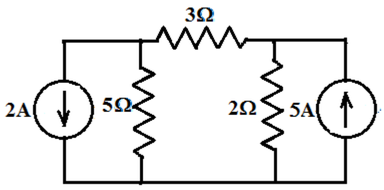
$$V = IR$$

$$I = V/R$$

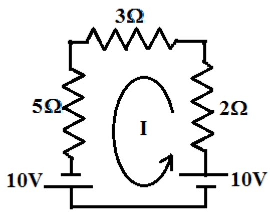
Hence, a current source I with a shunt resistance R can be replaced by a voltage source $V = IR$ in series with a resistance, R whereas a voltage source V with a series resistance R can be replaced by a current source$I = V/R$ in with a shunt resistance, R.

- A voltage source V_s in series with a resistance R_s may be converted into a current source $I_s = V_s/R_s$ in parallel with the same resistance and vice versa.
- Several voltage sources $(V_{s1}, V_{s2}, V_{s3}, ..., V_{sn})$ connected in series may be replaced by a single voltage source, $V_s = V_{s1} + V_{s2} + \ldots V_{sn} = \sum_1^n V_{si}$.
- Several current sources $(I_{s1}, I_{s2}, I_{s3}, ..., I_{sn})$ connected in parallel may be replaced by a single current source, $I_s = I_{s1} + I_{s2} + \ldots I_{sn} = \sum_1^n I_{si}$.
- Voltage sources should not be connected in parallel unless they have identical potential.
- Current sources should not be connected in series unless they are identical.

Example 1.43 *In the circuit shown in Figure 1.38a determine the value of current flowing through* 3Ω *resistor.*



(a) Original circuit



(b) Current sources transformed to Voltage sources

Figure 1.38: Circuit diagrams of Example 1.43

Solution Convert the current sources to voltage sources (Figure 1.38b).
Value of current flowing through 3Ω resistor

$$I = \frac{10+10}{5+3+2} = \frac{20}{10} = 2A$$

1.7 Kirchhoff's current law

This law is based on the law of conservation of charge. The algebraic sum of charge within a system cannot change. Thus sum of their rates of change i.e. currents must add up to zero.

Statement: Algebraic sum of Currents meeting at a point is zero. That is at a node $\sum^n I_i = 0$ where I_i is the current flowing into $i-$th branch.

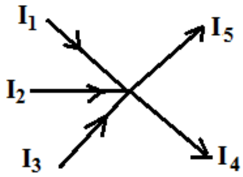


Figure 1.39: Currents meeting at a node

Considering the currents entering the node as positive and leaving the node as negative from the Figure 1.39 we can write,

$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$
$$\text{Or, } I_1 + I_2 + I_3 = I_4 + I_5$$

In other words, it may be stated that currents entering a node is equal to the currents leaving the node.

Example 1.44 *Using KCL determine the unknown current* I_1 *and* I_2 *in the circuit shown in Figure 1.40.*

Solution Applying KCL at nodes P and Q in Figure 1.40

$$20 = 6 + I_2 + 10$$
$$\text{Or, } I_2 = 4A$$
$$10 = 4 + I_1$$
$$\text{Or, } I_1 = 6A$$

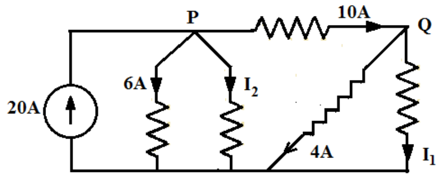


Figure 1.40: Circuit diagram for Example 1.44

1.8 Kirchhoff's voltage law

This law is based on the law of conservation of energy. The total work done in taking a positive charge around a closed path must add up to zero.

Statement: Algebraic sum of potential differences taken around a closed loop is zero. That is $\sum^n V_i$=0 where V_i is the potential drop across $i-$the component.
In other words, it may be stated that e.m.f.s around a closed loop is equal to the sum of voltage drops around the loop.

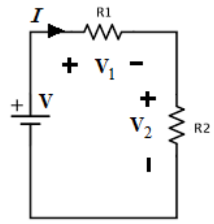


Figure 1.41: A closed circuit

For the closed circuit shown in Figure 1.41,

$$V - V_1 - V_2 = 0$$
$$\text{Or, } V = IR_1 + IR_2$$

Example 1.45 *Using KVL determine V_a in the circuit shown in Figure 1.42.*

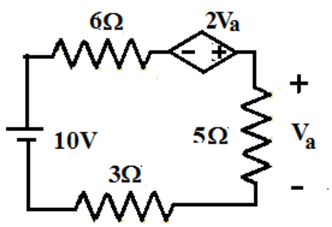


Figure 1.42: Circuit diagram for Example 1.45

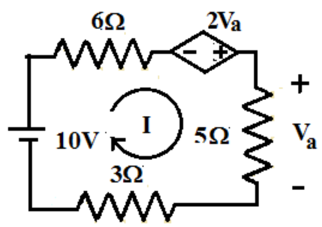


Figure 1.43: Circuit diagram for Example 1.45

Solution

$$V_a = 5I$$

Applying KVL in Figure 1.43

$$\begin{aligned} 10 + 2V_a &= (6+5+3)I = 14I = \frac{14}{5}V_a \\ \text{Or, } 50 &= 4V_a \\ \text{Or, } V_a &= \frac{50}{4} = 12.5V \end{aligned}$$

Example 1.46 *For the circuit shown in Figure 1.44, determine V_o.*

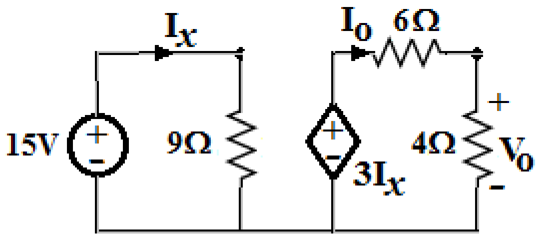


Figure 1.44: Circuit diagram for Example 1.46

Solution

$$\begin{aligned} I_x &= \frac{15}{9} = \frac{5}{3}\ A \\ 3I_x &= (6+4)I_o \\ \text{Or, } I_o &= \frac{3}{10}I_x = \frac{3}{10} \times \frac{5}{3} = 0.5\ A \\ V_o &= 4I_o = 4 \times 0.5 = 2\ volts \end{aligned}$$

Example 1.47 *In the network shown in Figure 1.45 determine the currents in each battery and in the* 6Ω *resistor.*

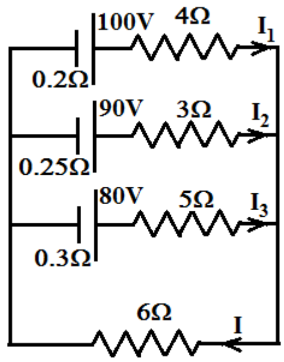


Figure 1.45: Circuit diagrams of Example 1.47

Solution

$$\begin{aligned} I_1 + I_2 + I_3 &= I \\ 10.2I_1 + 6I_2 + 6I_3 &= 100 \\ 6I_1 + 9.25I_2 + 6I_3 &= 90 \\ 6I_1 + 6I_2 + 11.3I_3 &= 80 \end{aligned}$$

Solving equations we get

$$\begin{aligned} I_1 &= 6.2136A \\ I_2 &= 4.9530A \\ I_3 &= 1.1504A \\ I &= I_1 + I_2 + I_3 = 12.3170A \end{aligned}$$

Example 1.48 *In the wheatstone bridge network shown in Figure 1.46a find the current through the galvanometer having a resistance of* 1000Ω

Solution From Fig.1.46b

$$\begin{aligned} 2200I_1 - 2000I_3 &= 4 \\ 830I_2 + 750I_3 &= 4 \\ 200I_1 - 80I_2 + 1000I_3 &= 0 \end{aligned}$$

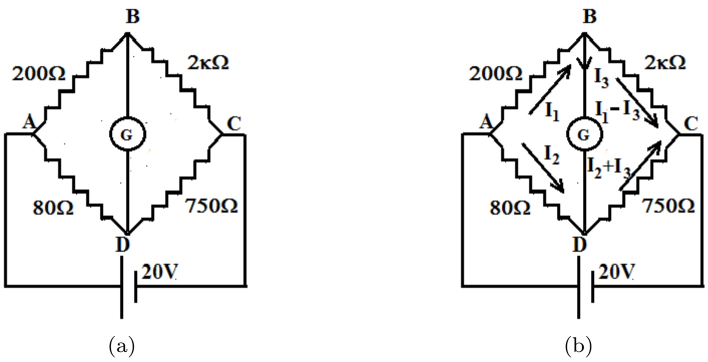


Figure 1.46: Circuit diagrams of Example 1.48

Solving equations we get

$$I_1 = 1.8341mA$$
$$I_2 = 4.8035A$$
$$I_3 = 17.5\mu A$$

Current flowing through the galvanometer $I_3 = 17.5\mu A$.

Example 1.49 *Find I_x and V_x in the network shown in Figure 1.47a.*

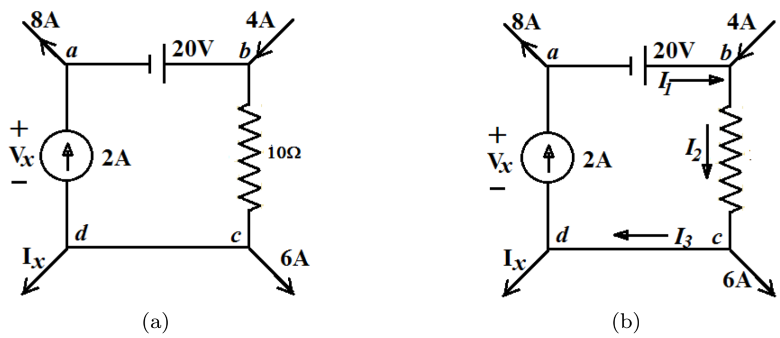


Figure 1.47: Circuit diagrams of Example 1.49

Solution Applying KCL to Figure 1.47b

$$\text{At 'a': } 8 + I_1 - 2 = 0 \qquad \text{Or, } I_1 = -6A$$
$$\text{At 'b': } 4 + I_1 - I_2 = 0 \qquad \text{Or, } I_2 = -2A$$
$$\text{At 'c': } 6 - I_2 + I_3 = 0 \qquad \text{Or, } I_3 = -8A$$
$$\text{At 'd': } 2 + I_x - I_3 = 0 \qquad \text{Or, } I_x = -10A$$
$$\text{Applying KVL: } V_x - 10I_2 + 20 = 0 \qquad \text{Or, } V_x = 40V$$

Example 1.50 *Find V_1 and V_2 in the network shown in Figure 1.48.*

Solution Applying KVL to Figure 1.48b

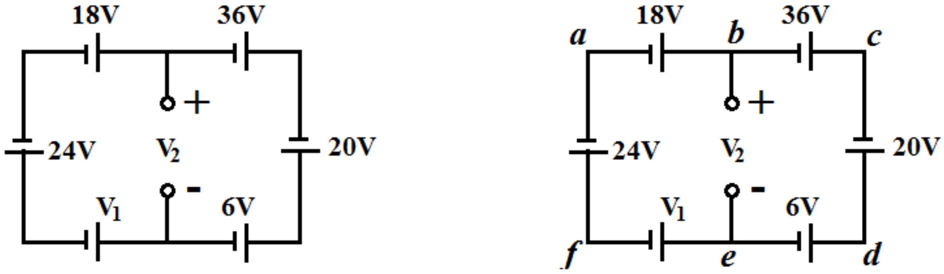
From loop a-c-d-f:

$$18 + 36 - 20 - 6 - V_1 - 24 = 0 \qquad \text{Or, } V_1 = 4V$$

From loop a-b-e-f:

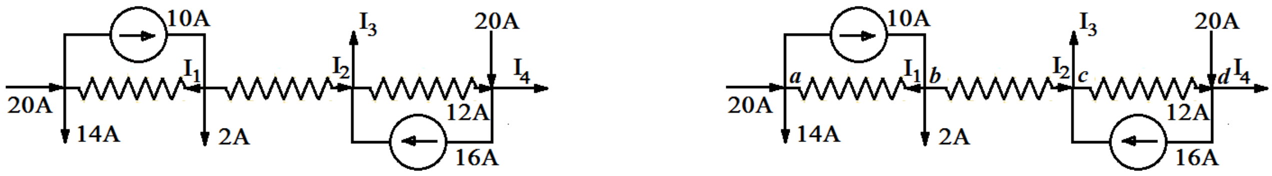
$$18 - V_1 - V_2 - 24 = 0 \qquad \text{Or, } V_2 = -10V$$

Example 1.51 *Find the values of unknown currents I_1, I_2, I_3 and I_4 of the network shown in Figure 1.49a.*



(a) (b)

Figure 1.48: Circuit diagrams of Example 1.50



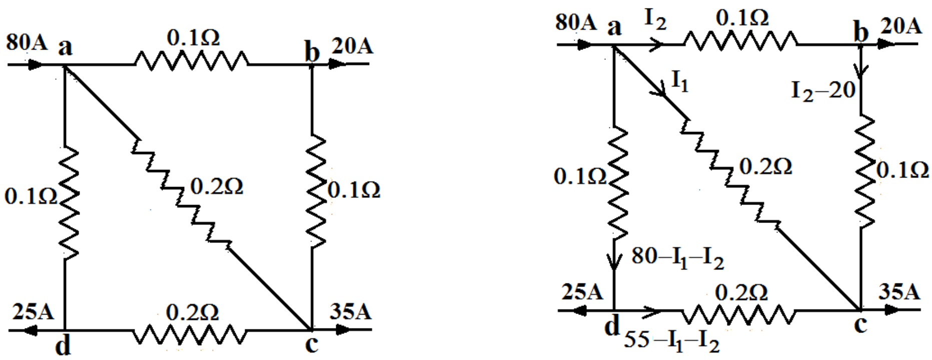
(a) (b)

Figure 1.49: Circuit diagrams of Example 1.51

Solution Applying KCL to Figure 1.49b

At 'a': $20 + I_1 - 10 - 14 = 0$	Or, $I_1 = 4A$
At 'b': $2 + I_1 + I_2 - 10 = 0$	Or, $I_2 = 4A$
At 'c': $12 - 16 - I_2 + I_3 = 0$	Or, $I_3 = 8A$
At 'd': $20 + 12 - 16 - I_4 = 0$	Or, $I_4 = 16A$

Example 1.52 *Find the current in branch ac for the network shown in Figure 1.50a.*



(a) (b)

Figure 1.50: Circuit diagrams of Example 1.52

Solution Let the current in Figure 1.50b through branch 'ac' be I_1 and 'ab' be I_2. Applying KCL at nodes 'a','d' and 'c' we get branch currents

$$I_{bc} = I_2 - 20$$
$$I_{ad} = 80 - I_1 - I_2$$
$$I_{dc} = I_{ad} - 25 = 55 - I_1 - I_2$$

Applying KVL to Figure 1.50b

$$\begin{aligned}
\text{At 'a-b-c':}\ & -0.2I_1 + 0.1I_2 + 0.1(I_2 - 20) = 0\\
\text{Or,}\ & -0.2I_1 + 0.2I_2 = 2\\
\text{Or,}\ & -I_1 + I_2 = 10\\
\text{At 'a-d-c':}\ & -0.2I_1 + 0.2(55 - I_1 - I_2) + 0.1(80 - I_1 - I_2) = 0\\
\text{Or,}\ & 0.5I_1 + 0.3I_2 = 19\\
& I_2 = 30A\\
& I_1 = I_{ac} = 20A
\end{aligned}$$

Example 1.53 *Find I and V_{AB} in the network shown in Figure 1.51a.*

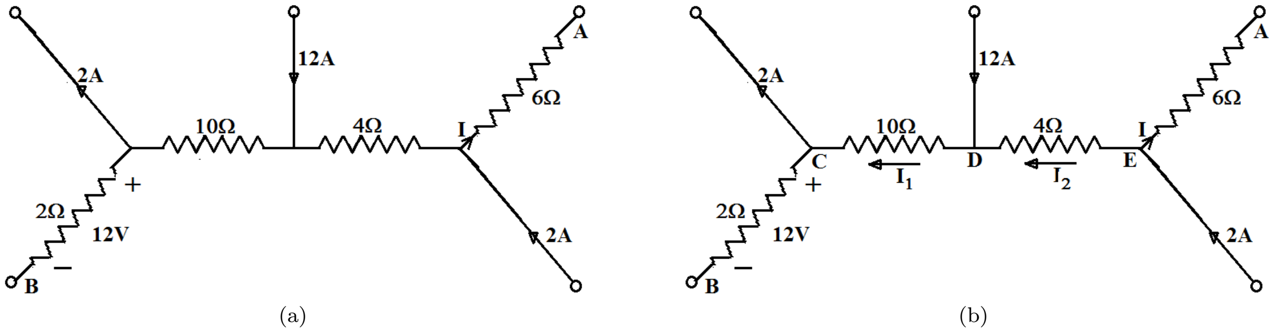


Figure 1.51: Circuit diagrams of Example 1.53

Solution Applying KCL to Figure 1.51b

At 'C': $2 + 6 - I_1 = 0$	Or, $I_1 = 8A$
At 'D': $12 - I_1 + I_2 = 0$	Or, $I_2 = -4A$
At 'E': $2 + 4 - I = 0$	Or, $I_2 = 6A$
Applying KVL: $V_{AB} + 36 + 16 - 80 - 12 = 0$	Or, $V_{AB} = 60V$

Example 1.54 *Find the voltage between A and B in the network shown in Figure 1.52a.*

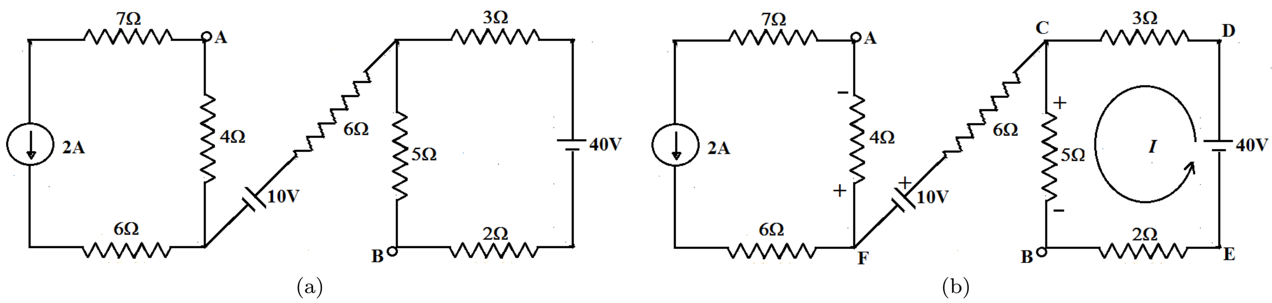


Figure 1.52: Circuit diagrams of Example 1.54

Solution From Figure 1.52b

$$I = \frac{40}{2 + 3 + 5} = 4A$$
$$V_{AB} = -2 \times 4 - 10 + 4 \times 5 = 2V$$

Example 1.55 *Find the voltage between A and B in the network shown in Figure 1.53a.*

Solution

$$I_1 = \frac{30}{4 + 6 + 10} = 1.5A$$
$$I_2 = \frac{60}{8 + 10 + 12} = 2A$$
$$V_{AB} = 12 \times 2 + 6 + (6 + 4) \times 1.5 = 45V$$

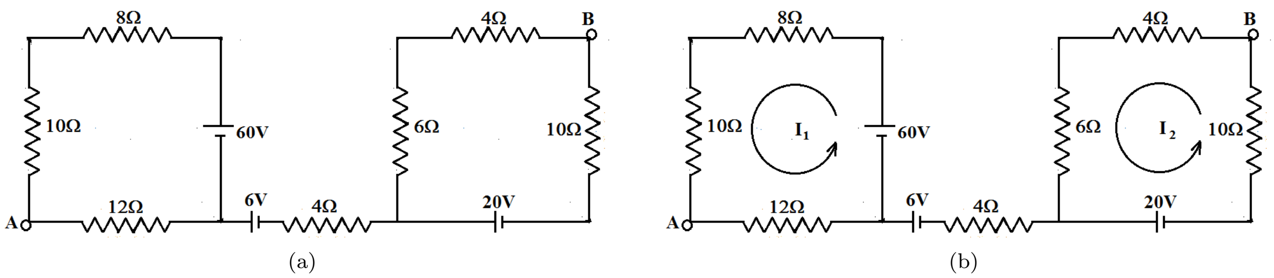


Figure 1.53: Circuit diagrams of Example 1.55