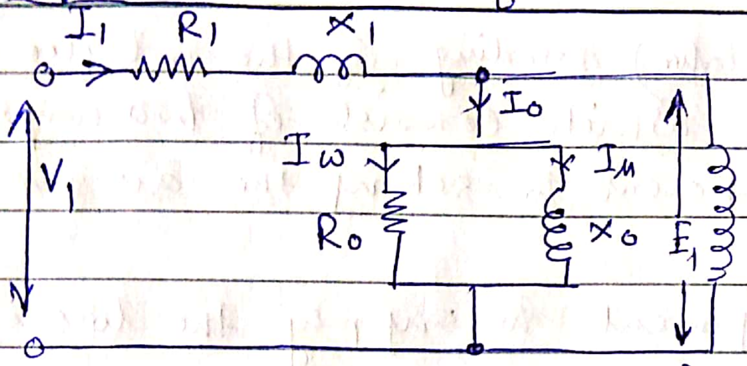
DEE - Sem 4 Class-note.

Equivalent ckt of an Induction Motor;



Stator model of an Induction Motor.

× [Let V_p be the phase voltage applied to the stator winding, E_1 be the emf per phase induced in the stator windings, I_1 the stator phase current, & $Z_1 = (R_1 + jX_1)$ be the impedance consisting of per phase resistance R_1 and leakage reactance per phase X_1 of the stator.

$\therefore V_p = E_1 + I_1(R_1 + jX_1) = E_1 + I_1 Z_1$

If the voltage drop in the stator impedance is ignored, then $V_p = E_1$

$V_1 = E_1 = 4.44 K_{w1} f T_1 \phi$ volt.

where, $K_{w1} \rightarrow$ winding factor of the stator winding

$f \rightarrow$ supply frequency

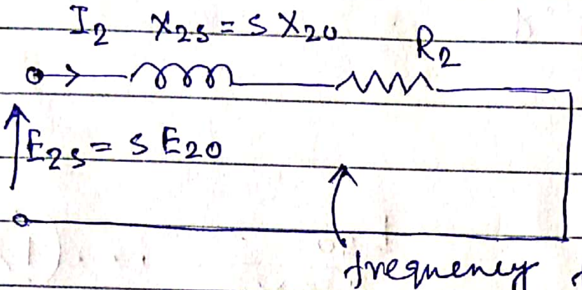
$T_1 \rightarrow$ stator number of turns in series per phase

$\phi \rightarrow$ air gap flux per pole.]

At no load, the primary (stator) winding of the induction motor draws a curren I_0 which consists of two compo
I_μ, the magnetising component to set up the air gap flux &
I_w, the core-loss component to supply the losses.
$\therefore I_0 = I_\mu + I_w$.

Due to the higher reluctance caused by the air gap of the induction motor, the magnetising reactance X_0 in an induction motor will have much smaller value. In a transformer, I_0 is about 2-5% of the rated current while in an induction motor it is approximately 25 to 40% of the rated current depending upon the size of the motor.

Rotor Circuit Model.



When a 3ϕ supply is applied to the stator winding, a voltage is induced in the rotor windings of the machine. In general, the greater the relative motion of the rotor and the stator magnetic field, the greater the resulting rotor voltage. The largest relative motion occurs when the rotor is stationary. This condition is called the stand-still condition. This is also known as the locked-rotor or blocked-rotor condition. If the induced rotor voltage at this condition is E_{20} then the induced voltage at any slip is given by $E_{2s} = sE_{20}$

The rotor resistance R_2 is a constant (except for the skin effect).

$X_2 = 2\pi f_2 L_2$ (L_2 = inductance of rotor)

But $f_2 = sf_1$ $\therefore X_{2s} = 2\pi s f_1 \cancel{f}L_2$

$= s(2\pi f_1 L_2) = sX_{20}$

X_{20} = leakage reactance of the rotor winding per phase when the rotor is at standstill.

X_{2s} = " " " " when the rotor is rotating at a slip s.

The rotor impedance is given by

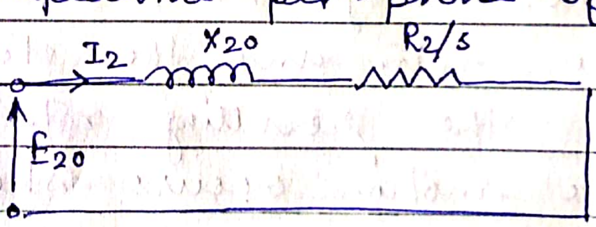
$Z_{2s} = R_2 + j\, X_{2s}$

$Z_{2s} = R_s + j\, s\, X_{20}$

$$I_2 \text{ (rotor current)} = \frac{E_{2s}}{Z_{2s}} = \frac{s E_{20}}{R_2 + j\, s\, X_{20}} \quad \ldots (i)$$

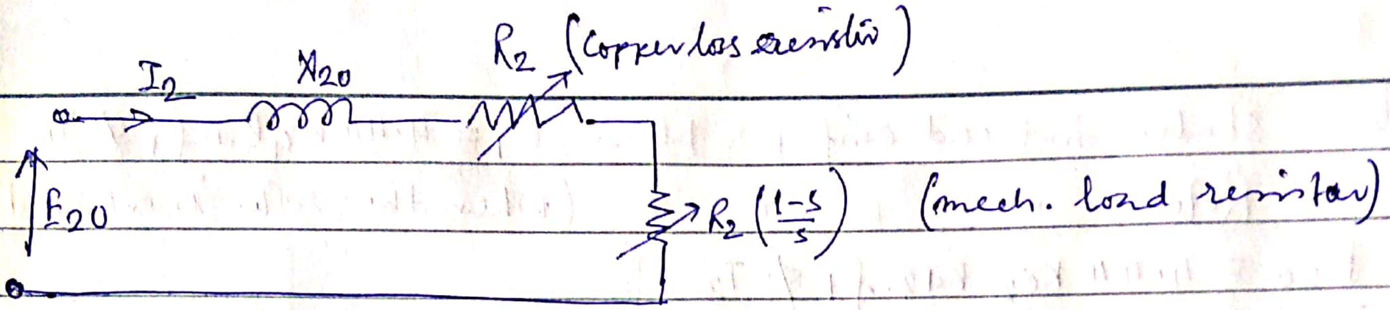
I_2 is a slip-frequency current produced by a slip-frequency induced voltage sE_{20} acting in the rotor ckt having an impedance per-phase of $(R_2 + j\, s\, X_{20})$

$$I_2 = \frac{E_{20}}{\frac{R_2}{s} + j\, X_{20}} \quad \ldots (ii)$$



There is a significant difference between Eqn (i) & (ii)
In eqn (ii) I_2 is considered to be produced by a constant line-frequency voltage E_{20} acting in a rotor ckt having an impedance per phase of $\left(\frac{R_2}{s} + j\, X_{20}\right)$.
Hence, the I_2 of Eqn (ii) is a line-frequency current, while I_2 of Eqn (i) is slip-frequency current.

The variable resistance $\frac{R_2}{s}$ may be replaced by the actual rotor winding resistance R_2 and a variable resistance R_{mech} which represents the mechanical shaft load. i.e. $R_{mech} = \frac{R_2}{s}(1-s)$



A complete circuit model (Equivalent ckt) referred to stator

In order to obtain the complete per-phase equivalent ckt for an induction motor, it is necessary to refer the rotor part of the model to the stator ckt frequency and voltage level.

In an ordinary transformer, the voltage, current and impedances on the secondary side can be transferred to the primary side by means of the turns ratio n of the transformer.

$$E_1 = E_2' = n E_2$$

$$I_1 = I_2' = \frac{I_2}{n}, \qquad Z_2' = n^2 Z_2$$

~~$Z_2' = \frac{Z_2}{n^2}$~~

Similar transformation can be done for induction motors' rotor ckt,

if, a_{eff} = effective turns ratio of the induction motor.
R_2' = resistance of the rotor winding per phase referred to the stator side.
X_{20}' = standstill rotor resistance per phase referred to the stator side.

~~For~~ stator induced emf per phase, $E_1 = 4.44\, K_{c1} K_{d1} f_1 \phi T_1$

Rotor " " " " (when the rotor is at standstill)

$E_{20} = 4.44\, K_{c2}\, K_{d2}\, f_1\, \phi\, T_2$

f_1 = stator frequency (supply frequency)

K_{c1} = pitch factor or coil span factor of stator winding

K_{c2} = " " " " " rotor "

K_{d1} = distribution factor of stator winding.

K_{d2} = " " " rotor " .

Induced emf per phase in the rotor when the rotor is rotating at slip s

$E_{2s} = s\, E_{20}$

$E_{2s} = 4.44\, K_{c2}\, K_{d2}\, s f_1\, \phi\, T_2$

Let, $K_{c1} K_{d1}$ = K_{w1}, winding factor of stator

$K_{c2} K_{d2}$ = K_{w2}, " " " rotor

$E_1 = 4.44\, K_{w1} f_1 \phi T_1$

$E_{2s} = 4.44\, K_{w2}\, s f_1\, \phi\, T_2$

$T_{e1} = K_{w1}\, T_1$

$T_{e2} = K_{w2}\, T_2$

$\therefore \dfrac{E_1}{E_{20}} = \dfrac{K_{w1} T_1}{K_{w2} T_2} = \dfrac{T_{e1}}{T_{e2}} = a_{eff}$

where, T_{e1} & T_{e2} are called effective stator & rotor turns per phase respectively. a_{eff} = effective turns ratio of an induction motor.

$$\frac{I_1}{I_2} = \frac{T_{e2}}{T_{e1}} = \frac{1}{a_{eff}}.$$

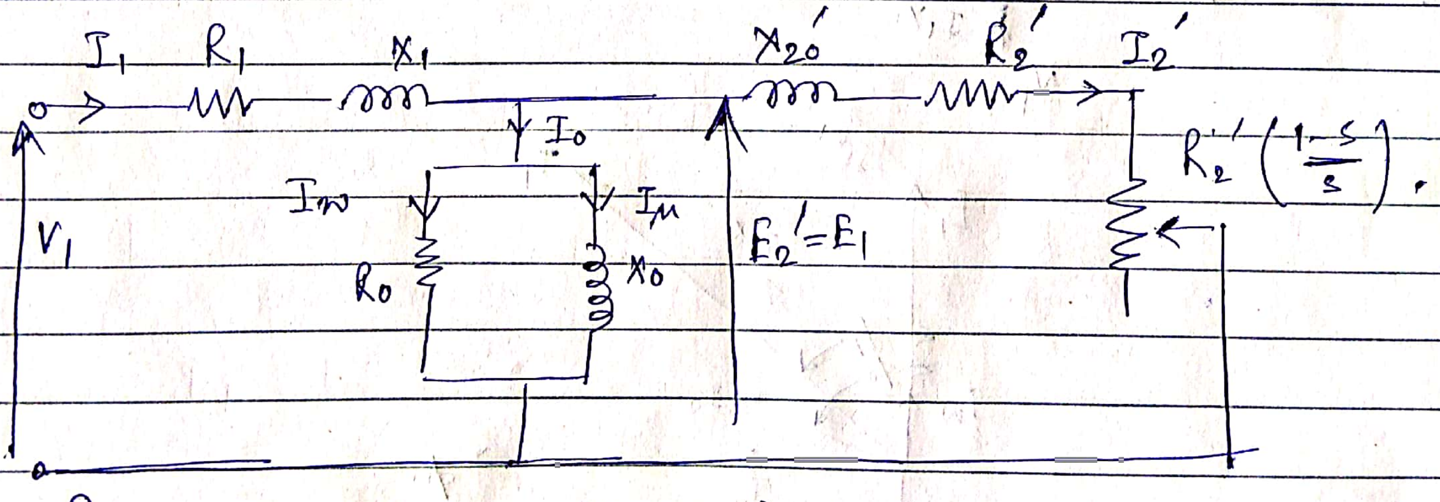
$$\frac{I_2'}{I_2} = \frac{T_{e2}}{T_{e1}} = \frac{1}{a_{eff}}.$$

$$\frac{E_2}{T_{e2}} = \frac{E_2'}{T_{e1}} \quad / \quad E_2' = \frac{T_{e1}}{T_{e2}} \cdot E_2 = a_{eff} E_2 = E_1$$

Similarly, $I_2' = \frac{T_{e2}}{T_{e1}} I_2 = \frac{I_2}{a_{eff}} = I_1$

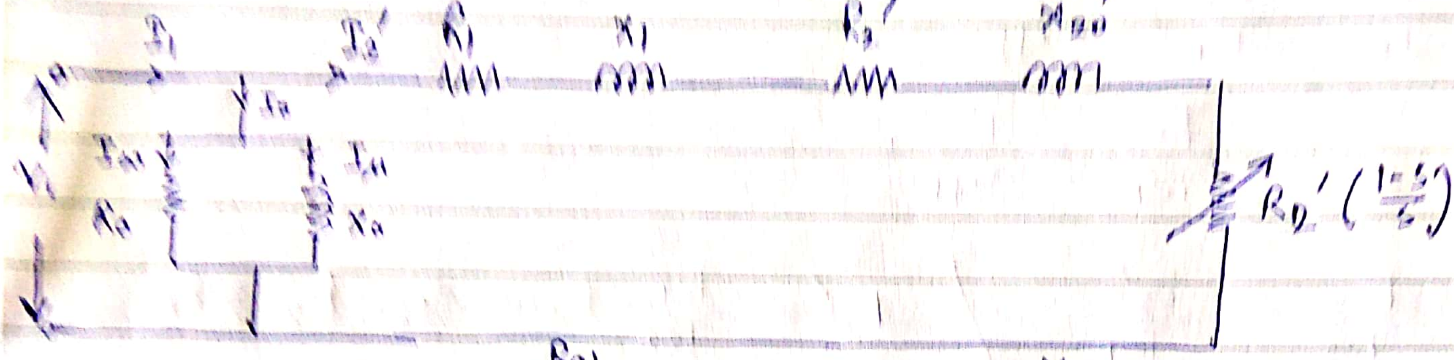
$$Z_{20}' = a_{eff}^2 \left(\frac{R_2}{s} + j X_{20} \right)$$

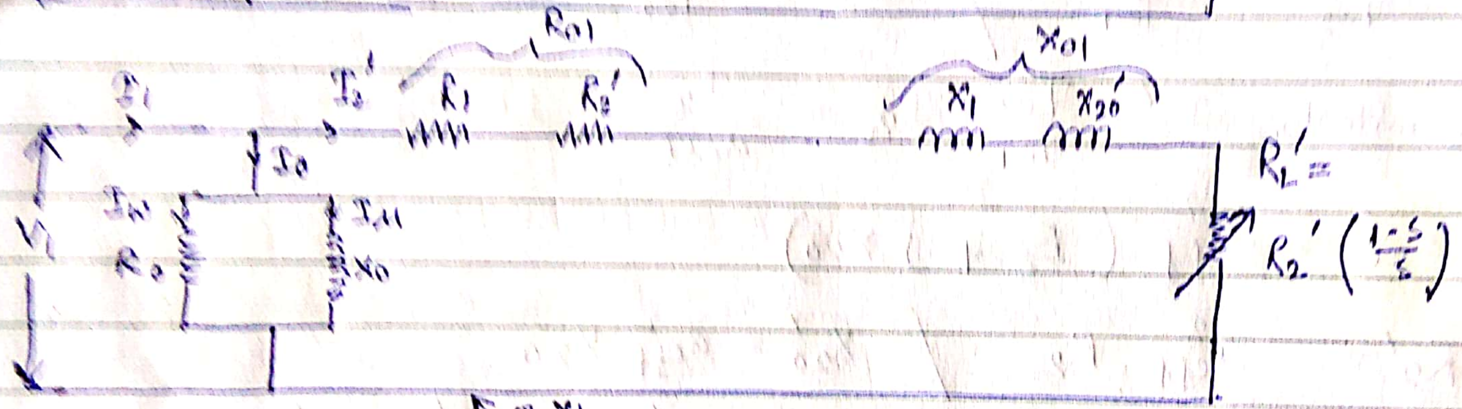
$$R_2' = a_{eff}^2 R_2 \quad / \quad X_{20}' = a_{eff}^2 X_{20}$$

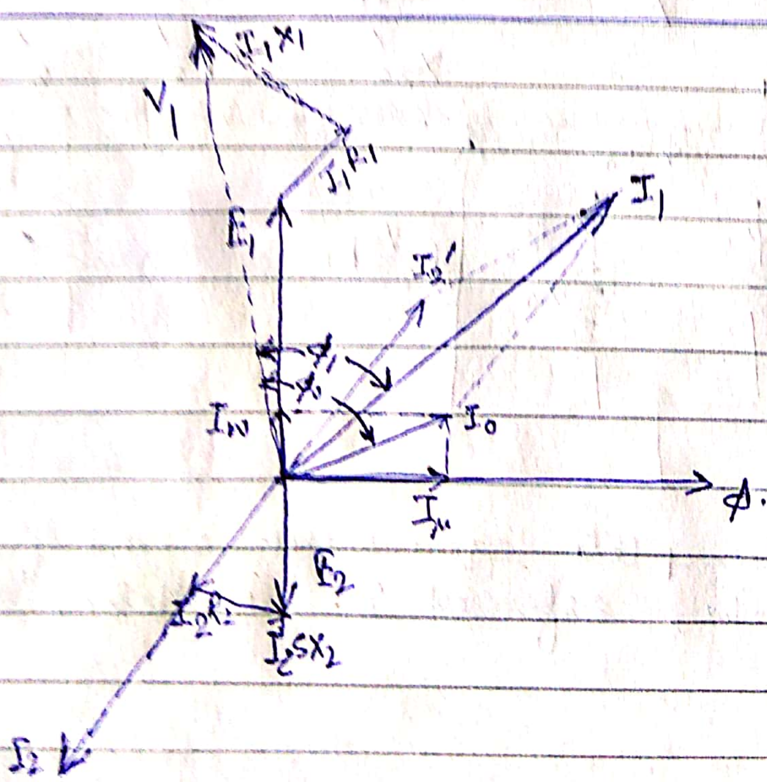


Per phase complete equivalent ckt of the induction motor referred to the stator.

Approximate Equivalent ckt.







Rotor Current

a) **Stand still condition:**

Let, E_{20} = emf induced per phase of the rotor at standstill

R_2 = resistance per phase of the rotor

X_{20} = reactance per phase of the rotor at stand still

Z_{20} = rotor impedance per phase at stand still

I_{20} = rotor current per phase at stand still

$Z_{20} = R_2 + jX_{20}$

$I_{20} = \frac{E_{20}}{Z_{20}}$.

P.f. at stand still, $\cos\phi_{20} = \frac{R_2}{Z_{20}} = \frac{R_2}{\sqrt{R_2^2 + X_{20}^2}}$

b) **Rotor current at slip s:**

Induced emf per phase in the rotor winding at slip s is

$E_{2s} = s\,E_{20}$

Rotor winding resistance per phase $= R_2$

Rotor winding ~~resistance~~ per phase at slip s is

$X_{2s} = 2\pi f_2 L = 2\pi (sf_1) L = sX_{20}$

$Z_{2s} = R_2 + jX_{2s} = R_2 + js\,X_{20}$

Rotor current at slip s; $I_{2s} = \frac{E_{2s}}{Z_{2s}}$.

P.f. at slip s, $\cos\phi_{2s} = \frac{R_2}{Z_{2s}}$.

* A 3-ϕ, 50Hz, 4-pole induction motor has a slip of 4%. Calculate (a) speed of the motor (b) frequency of the rotor emf.
If the rotor resistence of 1Ω and standstill reactance of 4Ω, calculate the P.f. (i) at standstill & (ii) at a speed of 1400 r.p.m.

$$N_s = \frac{120 f_1}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

$$s = 4\% = 0.04 \text{ p.u.}$$

(a) speed of the motor:

$$N = (1-s) N_s = 1440 \text{ rpm.}$$

(b) frequency of of the rotor emf

$$f_2 = s f_1 = 0.04 \times 50 = 2 \text{ Hz.}$$

(i) $R_2 = 1\Omega$, $X_{20} = 4\Omega$.

$$\therefore Z_{20} = 1 + j4 = 4.123 \angle 75.96^\circ \ \Omega$$

Power factor at standstill

$$\cos\phi_{20} = \cos 75.96^\circ = 0.2425 \text{ (lag)}$$

(ii) The slip at 1400 rpm,

$$s_1 = \frac{N_s - N}{N_s} = \frac{1500 - 1400}{1500} = \frac{1}{15}.$$

$$\therefore Z_{2s_1} = R_2 + j s_1 X_{20} = 1 + j \times \frac{1}{15} \times 4$$

$$= 1.03495 \angle 14.93^\circ$$

$$\therefore \cos\phi_{2s_1} = \cos 14.93^\circ = 0.9662 \text{ (lag)}.$$

*A 3ϕ slip-ring induction motor gives a reading of 60V across slip rings when at rest with normal stator voltage applied. The rotor is star connected and has an impedance of $(0.8+j6)\,\Omega$ per phase. Find the rotor current when the m/c is (i) at stand-still with the slip-rings joined to a star connected starter with a phase impedance of $(4+j3)\,\Omega$ & (b) running normally with a 5% slip.

$$E_{20} = \frac{60}{\sqrt{3}} = 34.64\ V$$ (emf induced per phase of the rotor at standstill)

Total impedance of the rotor at stand still = impedance of rotor + impedance of satter

$$= (0.8+j6) + (4+j3) = (4.8+j9)\,\Omega$$

(a) current at standstill, $I_{20} = \frac{E_{20}}{Z_{20}} = \frac{34.64\angle 0^\circ}{4.8+j8} = \frac{34.64\angle 0^\circ}{10.2\angle 61.93^\circ}$

$$= 3.396\angle -61.93^\circ\ A$$

(b) $s = 5\% = 0.05$ p.u **

$$I_{2s} = \frac{E_{2s}}{Z_{2s}} = \frac{sE_{20}}{R_2 + j s X_{20}} = \frac{0.05 \times 34.64}{0.8 + j\,0.05 \times 6}$$

$$= \frac{1.732}{0.8+j0.3} = \frac{1.732}{0.8544\angle 20.56^\circ} = 2.027\angle -20.56^\circ\ A$$

** During normal running, the starting resistance are cut-off.

Relation between Rotor copper loss and Rotor input.

Let, T_d = developed torque = torque exerted on the rotor by rotating flux

n_s = synchronous speed (r.p.s)

n_r = rotor speed (r.p.s).

Power transferred from stator to rotor = air-gap power P_g

$P_g = \omega_s T_d = \underline{2\pi n_s T_d}$ = input power to rotor.

Mechanical power developed by the rotor,

$P_{md} = \omega_r T_d = \underline{2\pi n_r T_d}$

$\therefore$ Total I^2R loss in rotor = Power transferred from stator to rotor − mechanical power developed by the rotor.

$\therefore P_{rc} = P_g - P_{md} = 2\pi T_d (n_s - n_r)$

$$\therefore \frac{\text{total } I^2R \text{ loss in rotor}}{\text{Input power to rotor}} = \frac{2\pi T_d (n_s - n_r)}{2\pi T_d \, n_s} = s$$

$\therefore$ rotor copper loss = $s \times$ rotor input.

$$P_{rc} = s P_g = s P_{ir}$$ [$s \times$ air gap power $\rightarrow$ slip power]

slip power − It is the portion of air-gap power which is not converted into mechanical power.

rotor input power = mechanical power developed + rotor copper loss.

$P_{ir} = P_{md} + P_{rc}$

$P_{rc} = s P_g = s(P_{rc} + P_{md})$

$\therefore P_{rc}(1-s) = s P_{md}$

$P_{rc} = \frac{s}{1-s} P_{md}$

$\therefore$ Rotor copper loss = $\left(\frac{s}{1-s}\right) \times$ mech. power developed by the rotor.

$\omega_r = 2\pi n_r$
$\omega_s = 2\pi n_s$

$\frac{n_s - n_r}{n_s} = s.$

$\frac{\omega_s - \omega_r}{\omega_s} = s$ / $\omega_s - \omega_r = s\omega_s$
$\omega_r = \omega_s(1-s)$ *

$\therefore P_g : P_{rc} : P_{md} = 1 : s : (1-s).$

Once the air-gap Power P_g is determined, three quantities may be found from the slip & synchronous speed.

$P_{rc} = sP_g$

$P_{md} = (1-s)P_g$

$T_d = \frac{P_g}{\omega_s}$

$$\left[P_{rc} = \frac{s}{(1-s)} \cdot P_{md} \right]$$

$$P_{md} = \frac{(1-s)}{s} P_{rc}$$
$$= \frac{(1-s)}{s} \cdot s \cdot P_g$$
$$= (1-s) P_g$$

Developed Torque T_d

The developed torque or induced torque in a m/c is defined as the torque generated by the internal electric-to-mechanical power conversion. The torque is called the electromagnetic torque.

$$T_d \text{ (developed torque)} = \frac{\text{mech. power developed}}{\text{mech. angular velocity of the rotor}}$$

$$T_d = \frac{P_{md}}{\omega_r}$$

Since $P_{md} = (1-s)P_g$ and $\omega_r = (1-s)\omega_s$ *

$$\therefore T_d = \frac{(1-s)P_g}{(1-s)\omega_s} = \frac{P_g}{\omega_s}.$$ *

The Power-flow diagram of an Induction Motor

Stator input ($\sqrt{3}\, V_L I_L \cos\phi$)

- I^2R loss in Stator
- Core loss in stator
- Stator output (or rotor input) (air gap power)
  - Total mechanical power developed
    - Rotor core loss
    - friction loss
    - windage loss
    - stray loss
    - useful power output obtained at rotor shaft.
  - I^2R loss in Rotor

Torque in Induction motor

Rotor input $= E_2 I_2 \cos\phi_2$

where, E_2 = rotor emf

I_2 = rotor current

$\cos\phi_2$ = rotor power factor

$\therefore$ $T \propto E_2 I_2 \cos\phi_2$,

$$T \propto E_2 \times \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

where, $I_2 = \frac{sE_2}{Z_2}$ & $\cos\phi_2 = \frac{R_2}{Z_2}$

& $Z_2 = \sqrt{R_2^2 + (sX_2)^2}$

& s is the slip at Torque T.

$$T \propto \frac{S E_2^2 R_2}{R_2^2 + (S X_2)^2}$$

as E_2 is proportional to V_1 $\therefore$ $T \propto V_1^2 \propto (\text{supply Voltage})^2$

Starting Torque: (T_{st}), at starting, $S = 1$

$\therefore$ $T_{starting} = \dfrac{K E_2^2 R_2}{R_2^2 + X_2^2}$ [The value of K may be taken as $\dfrac{3}{2\pi n_s}$]

Maximum Torque: (T_{max})

$T = \dfrac{K S E_2^2 R_2}{R_2^2 + (S X_2)^2}$, for max. torque denominator of expression must be minimum.

$R_2^2 + (S X_2)^2 = (R_2 - S X_2)^2 + 2 R_2 S X_2$.

Since, $2 \times R_2 \times S X_2$ can not be zero, $(R_2 - S X_2)^2 = 0$

or, $\boxed{R_2 = S X_2}$ * putting the value of R_2 in the torque expression

$$T_{max} = \frac{K S E_2^2 R_2}{R^2 + (S X_2)^2} = \frac{K S E_2^2 \times (S X_2)}{2 S^2 X_2^2} = \frac{K E_2^2}{2 X_2} = \frac{3}{2\pi n_s} \cdot \frac{E_2^2}{2 X_2}$$

* Corresponding to maximum torque, $S = \dfrac{R_2}{X_2}$.

This value is specially termed as 'a', $\therefore$ $a = \dfrac{R_2}{X_2}$

The ~~fton~~ following points should be noted:

1) Maximum torque is independent of rotor resistance.
2) Slip corresponding to maximum torque depends upon rotor resistance
3) T_{max} varies inversely as rotor reactance
4) maximum starting torque will occurs when $R_2 = S X_2$

Full load Torque: (T_f)

If full load slip is S_f, then $T_f = \dfrac{K S_f E_2^2 R_2}{R_2^2 + (S_f X_2)^2}$

Relation between T_{st}, T_{max} & T_f ;

i) $$\frac{T_f}{T_{max}} = \frac{k S_f E_2^2 R_2}{R_2^2 + (S_f X_2)^2} \div \frac{k E_2^2}{2 X_2} = \frac{K E_2^2 S_f R_2}{R_2^2 + (S_f X_2)^2} \times \frac{2 X_2}{K E_2^2}$$

$$= \frac{2 S_f R_2 X_2}{R_2^2 + S_f^2 X_2^2}$$

$$= \frac{\frac{2 S_f R_2 X_2}{X_2^2}}{\frac{R_2^2 + S_f^2 X_2^2}{X_2^2}} = \frac{2 a S_f}{a^2 + S_f^2} . \quad \left[\frac{R_2}{X_2} = a\right]$$

(ii) $$\frac{T_s}{T_{max}} = \frac{K E_2^2 R_2}{R_2^2 + X_2^2} \div \frac{K E_2^2}{2 X_2}$$

$$= \frac{K E_2^2 R_2}{R_2 + X_2^2} \times \frac{2 X_2}{K E_2^2} = \frac{2 R_2 X_2}{R_2^2 + X_2^2} = \frac{\frac{2 R_2 X_2}{X_2^2}}{\frac{R_2^2 + X_2^2}{X_2^2}}$$

$$= \frac{2a}{1 + a^2}$$

Torque Slip characteristics

$$T = \frac{K S E_2^2 R_2}{R_2^2 + (S X_2)^2} .$$

i) At $S = 0$, $T = 0$. Therefore torque-slip curve starts from zero value

(ii) when slip is small, SX_2 is negligible in ~~comparesion~~ comparison to R_2, $\therefore$ $T \propto \frac{S}{R}$

or, $T \propto S$ or $T = K_1 S$ [R = constant]

So, for this portion torque-slip curve will be straight.

(iii) At low speed & at starting s approaches unity. also $a\left(=\frac{R_2}{X_2}\right)$ is usually small in normal motors, at typical value being $\frac{R_2}{X_2} = 0.2$ & R_2 is neglected in comparison to sX_2.

$\therefore$ $T \propto \frac{s}{s^2 X_2^2}$ or $T \propto \frac{1}{s}$ [$\because$ X_2 is constant].

or $T = \frac{K_2}{s}$ (Torque-Slip curve is a rectangular hyperbola)

(iv) At the value of $s = \frac{R_2}{X_2}$, the torque is maximum.