① Revision of formulas of Differentiations before starting Partial Derivative:

(1) $\frac{d}{dx}(x^n) = nx^{n-1}$

(2) $\frac{d}{dx}(x) = 1$

(3) $\frac{d}{dx}(e^x) = e^x$

(4) $\frac{d}{dx}(a^x) = a^x \cdot \log_e a$

(5) $\frac{d}{dx}(\log x) = \frac{1}{x}$

(6) $\frac{d}{dx}(\sin x) = \cos x$

(7) $\frac{d}{dx}(\cos x) = -\sin x$

(8) $\frac{d}{dx}(\tan x) = \sec^2 x$

(9) $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

(10) $\frac{d}{dx}(\sec x) = \sec x \tan x$

(11) $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

(12) $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

(13) $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

(14) $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

(15) $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

(16) $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$

(17) $\frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$

② Partial Derivative

You are familiar with the ordinary derivative $\frac{df}{dx}$ from single variable calculus. Now we will discuss about the Partial Derivative of the function of two or more ~~variables of~~ independent variables (one does not depend on other).

For example, $z = f(x, y) = x^2 - 2xy$. Here $x^2 - 2xy$ is a function of x and y.

Now, how to differentiate the function $f(x, y)$ ~~with~~ partially with respect to x.

$$f(x, y) = x^2 - 2xy$$

$$f_x(x, y) = \frac{\partial f}{\partial x} = 2x - 2y \quad \left[\text{diff. the fu. } f \text{ with respect to } x, \text{ taking } y \text{ as a constant}\right]$$

[$\frac{\partial f}{\partial x}$ also denoted by $f_x(x, y)$. It will be $\frac{\partial f}{\partial x}$, not $\frac{df}{dx}$]

③ NOW how to differentiate the fu. f with respect to y (taking x as a constant)

$$f(x,y) = x^2 - 2xy$$

$$f_y(x,y) = \frac{\partial f}{\partial y} = -2x \quad \text{[taking } x \text{ as a contant]}$$

[Explanation:- $\frac{\partial}{\partial y}(x^2 - 2xy) = \frac{\partial}{\partial y}(x^2) - \frac{\partial}{\partial y}(2xy)$

$= 0 - 2x.1 \quad (\because x = \text{const})$

$= -2x$]

Second order partial Derivative

$$\frac{\partial}{\partial x}\left(f_x(x,y)\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx}(x,y)$$

Here, $f_x(x,y) = 2x - 2y$

$$\therefore \quad f_{xx}(x,y) = \frac{\partial}{\partial x}\left(f_x(x,y)\right) = \frac{\partial}{\partial x}(2x - 2y)$$
$$= 2$$

Similarly we can find $f_{yy}(x,y)$ means $\frac{\partial}{\partial y}\left(f_y(x,y)\right)$

~~NOW~~, NOW, $\frac{\partial}{\partial x}\left(f_y(x,y)\right) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = f_{xy}$

and $\frac{\partial}{\partial y}\left(f_x(x,y)\right) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x} = f_{yx}$

(4) For example :—

$$f(x, y) = x^3 y + 3y^2 x$$

NOW, $f_x(x, y) = \frac{\partial f}{\partial x} = 3x^2 y + 3y^2$ (diff with respect to x, y cont)

$f_y(x, y) = \frac{\partial f}{\partial y} = x^3 + 6yx$ (diff. w.r. to y, x cont)

$$f_{xx}(x, y) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left(3x^2 y + 3y^2\right)$$

$= 6xy$ [diff. w.r. to x, y cont]

$$f_{yy}(x, y) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}\left(x^3 + 6yx\right)$$

$= 6x$ [diff. w.r. to y, x cont]

$$f_{xy}(x, y) = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}\left(x^3 + 6yx\right)$$

$= 3x^2 + 6y$ [diff. w.r. to x, y cont]

$$f_{yx}(x, y) = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial y}\left(3x^2 y + 3y^2\right)$$

$= 3x^2 + 6y$ [diff. w.r. to y, x cont]